Quark and lepton masses at the GUT scale

Martin Spinrath

Max Planck Institut für Physik

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Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

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What are Yukawa couplings?

Yukawa couplings are the couplings between fermions and Higgs fields. In the SM:

$$\mathcal{L}_{\text{Yuk}} = -y_b \overline{b}_R Q_L \epsilon H + \text{h.c.}$$

After electroweak symmetry breaking:

$$\begin{aligned} \mathcal{L}_{\text{Yuk}} &= -y_b \overline{b}_R \begin{pmatrix} t_L \\ b_L \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} v + H^0 \\ H^- \end{pmatrix} + \text{h.c.} \\ &\to -y_b v \overline{b}_R b_L + \ldots + \text{h.c.} \end{aligned}$$

Compare to a mass term in the Lagrangian density:

$$\mathcal{L}_{\text{mass}} = -m_b \overline{b}_R b_L + \text{h.c.}$$

But we have three generations?!

It is easy to extend this to three generations ($\rightarrow$ 1/2 Nobelprize 2008):

$$\mathcal{L}_{\text{Yuk}} = -(y_b)_{ij} \left(\overline{d}_R\right)_i (Q_L)_j \, \epsilon H + (y_t)_{ij} \left(\overline{t}_R\right)_i (Q_L)_j \, \epsilon H^c + \text{h.c.}$$

The CKM Matrix connects gauge with mass eigenstates.
In the SM we get 13 free parameters from the Yukawa couplings.

SUSY Yukawas?

In SUSY there have to be at least two Higgs doublets H_u and H_d.

$$\mathcal{L}_{\text{Yuk}} = -y_b \overline{b}_R Q_L \epsilon H_d - y_\tau \overline{\tau}_R L_L \epsilon H_d - y_t \overline{t}_R Q_L \epsilon H_u + \text{h.c.}$$

These doubletts have the vevs v_u and v_d with the ratio

$$\tan\beta = \frac{v_u}{v_d} \overset{?}{\approx} \frac{y_t v_u}{y_b v_d} = \frac{m_t}{m_b}$$

GUT Yukawas?

In GUTs the fermions sit in larger representations, e.g. in $SU(5)$:

$$\mathcal{L}_{\mathrm{Yuk}} = \frac{y}{\sqrt{2}} \overline{\begin{pmatrix} 0 & t^{c3} & -t^{c2} & t_1 & b_1 \\ -t^{c3} & 0 & t^{c1} & t_2 & b_2 \\ t^{c2} & -t^{c1} & 0 & t_3 & b_3 \\ -t_1 & -t_2 & -t_3 & 0 & \tau^+ \\ -b_1 & -b_2 & -b_3 & -\tau^+ & 0 \end{pmatrix}}_{ij} \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ \tau^+ \\ -\nu_\tau^c \end{pmatrix}_i \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ v \end{pmatrix}_j + \mathrm{h.c.}$$

$$\rightarrow -\frac{y}{\sqrt{2}} v \left(\overline{b} b + \overline{\tau} \tau \right) + \mathrm{h.c.}$$

Alternative with a 45-dim. Higgs field $\rightarrow m_\mu / m_s = 3$ [Georgi and Jarlskog, '79]

So what are the mass ratios at the GUT scale?

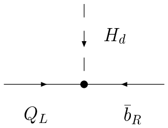
So what are the mass ratios at the GUT scale ($\approx 2 \cdot 10^{16}$ GeV)?

- Top-down approach: Prediction from theory $\rightarrow$ low-energy theory? $\rightarrow$ SUSY threshold effects necessary!
- Bottom-up approach: Extrapolate masses to GUT scale [Fusaoka and Koide ’98, Xing et. al. ’07] (no SUSY thresholds included) $\rightarrow$ Inclusion? [Antusch and Spinrath, arXiv:0804.0717 to appear in Phys. Rev. D]

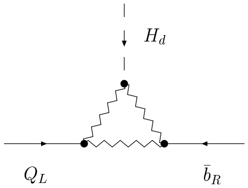
Outline

Yukawa and Feynman

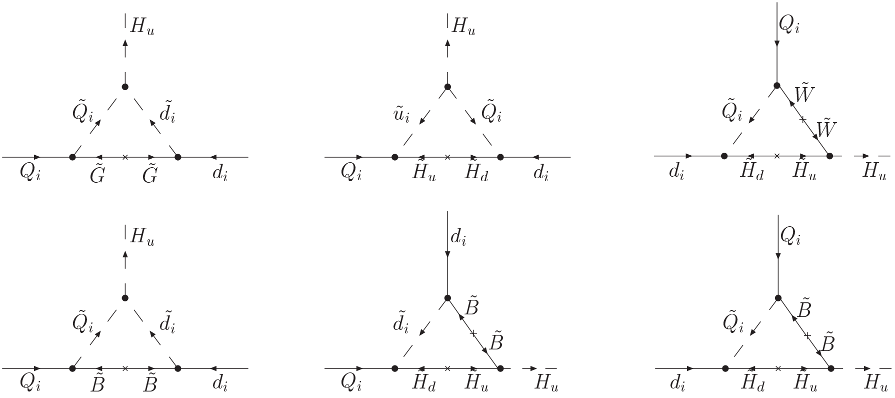
On tree level in the MSSM we have:



State of the art (at least) one loop:



"Wrong" Higgs coupling diagrammatic



"Wrong" Higgs coupling in the Lagrangian

This "wrong" Higgs couplings give $\tan\beta$ enhanced corrections to the down type masses:

$$\begin{aligned} \mathcal{L} &= -y_b \overline{b}_R Q_L \epsilon H_d + \delta\tilde{y}_b \overline{b}_R Q_L \epsilon H_u^c + \text{h.c.} \\ &\rightarrow -(y_b v_d + \delta\tilde{y}_b v_u) \overline{b}_R b_L + \text{h.c.} \\ &\Rightarrow m_b = m_b^{(0)} \left(1 + \frac{\delta\tilde{y}_b}{y_b} \tan\beta\right) \end{aligned}$$

For the up type quarks:

$$m_t = m_t^{(0)} \left(1 + \frac{\delta\tilde{y}_t}{y_t} \frac{1}{\tan\beta}\right)$$

Matching relations

How are the low-energy (non-SUSY) related to the high-energy (SUSY) Yukawa couplings?

For the down-type fermion Yukawa couplings:

$$y_i^{\mathrm{MSSM}} = \frac{y_i^{SM}}{\cos\beta(1+\epsilon_i \tan\beta)},$$

where $y_i^{SM} = m_i/v$, $\epsilon_i = \delta\tilde{y}_i/y_i$ and $i = d, s, b, e, \mu, \tau$.
For the up type quarks we use:

$$y_i^{\mathrm{MSSM}} = \frac{y_i^{SM}}{\sin\beta},$$

where $i = u, c, t$

Outline

Parameter sets

SUSY parameter in TeV	Case g_+	Case g_-	Case a
$m_{\tilde{f}}$	[0.5, 1.5]	[0.5, 1.5]	[0.5, 1.5]
M_1	[0.5, 1]	[0.5, 1]	[1.65, 3.3]
M_2	[1, 2]	[1, 2]	[0.5, 1]
M_3	[3, 6]	[3, 6]	[-9, -4.5]
μ	0.5	-0.5	0.5
A_t	[-1, 1]	[-1,1]	[-1,1]
M_{SUSY}	1	1	1

- The parameters are chosen in the TeV region such that we can use only one matching scale and neglect EW symmetry breaking effects.
- The cases $g_\pm$ are inspired by universal gaugino masses ($M_1 : M_2 : M_3 \approx 1 : 2 : 6$).
- Case a is inspired by anomaly-mediated SUSY breaking ($M_1 : M_2 : M_3 \approx 3.3 : 1 : -9$).

Impact of the sparticle spectrum

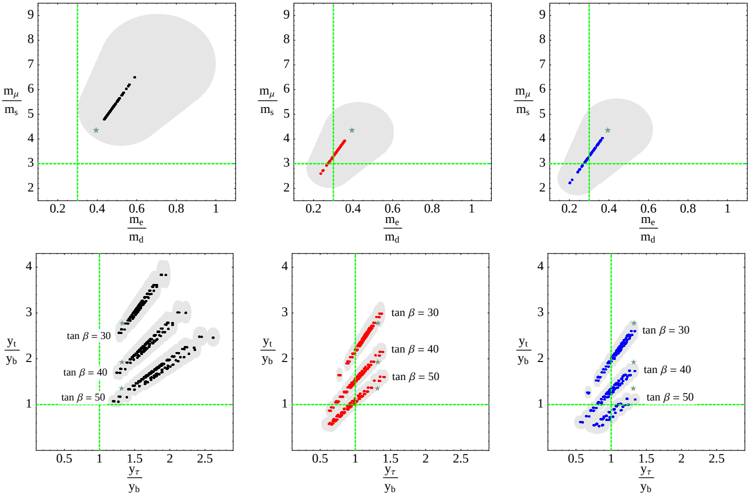


Figure: Scatter plots illustrating the ranges of GUT scale ratios of the masses resp. Yukawas for case g_+, g_- and a for $\tan\beta = 30$, 40 and 50, taken from [Antusch and Spinrath, arXiv:0804.0717].

Conclusions

- The SUSY threshold corrections have to be included for discussions of Yukawa couplings above the SUSY scale.
- The corrections can help to the realize the Georgi Jarlskog relations and complete third family Yukawa unification.
- The corrections also allow many other possible values for the mass and Yukawa coupling ratios at the GUT scale.

The End

Thanks for your attention

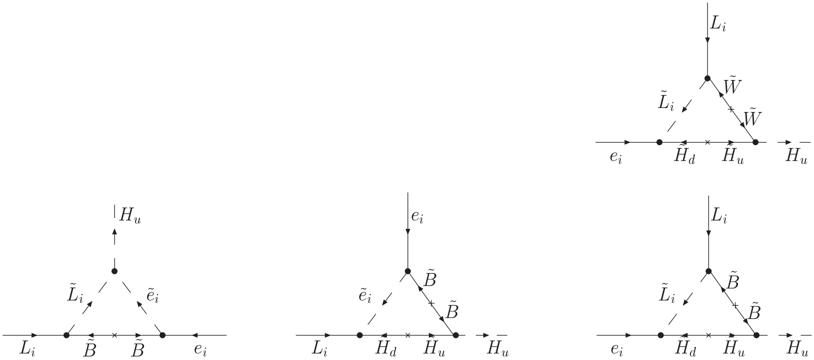


Figure: Feynman diagrams contributing to the $\tan\beta$ enhanced SUSY threshold corrections to charged lepton Yukawa couplings.

Backup: Down type quark SUSY threshold corrections

For the down type quarks we can decompose $\epsilon_i = \epsilon_i^G + \epsilon_i^B + \epsilon_i^W + \epsilon^y \delta_{ib}$ in the EW unbroken phase, with

$$\epsilon_i^G = -\frac{2\alpha_S}{3\pi}\frac{\mu}{M_3} H_2(u_{\tilde{Q}_i}, u_{\tilde{d}_i}) \,, \tag{1a}$$

$$\epsilon_i^B = \frac{1}{16\pi^2}\left[\frac{g'^2}{6}\frac{M_1}{\mu}\left(H_2(v_{\tilde{Q}_i}, x_1) + 2H_2(v_{\tilde{d}_i}, x_1)\right) + \frac{g'^2}{9}\frac{\mu}{M_1} H_2(w_{\tilde{Q}_i}, w_{\tilde{d}_i})\right] , \tag{1b}$$

$$\epsilon_i^W = \frac{1}{16\pi^2}\frac{3g^2}{2}\frac{M_2}{\mu} H_2(v_{\tilde{Q}_i}, x_2) \,, \tag{1c}$$

$$\epsilon^y = -\frac{y_t^2}{16\pi^2}\frac{A_t}{\mu} H_2(v_{\tilde{Q}_3}, v_{\tilde{u}_3}) \,, \tag{1d}$$

where $u_{\tilde{f}} = m_{\tilde{f}}^2/M_3^2$, $v_{\tilde{f}} = m_{\tilde{f}}^2/\mu^2$, $w_{\tilde{f}} = m_{\tilde{f}}^2/M_1^2$, $x_1 = M_1^2/\mu^2$ and $x_2 = M_2^2/\mu^2$ and where all mass parameters are assumed to be real.

For the charged leptons we can decompose $\epsilon_i = \epsilon_i^B + \epsilon_i^W$

$$\epsilon_i^B = \frac{1}{16\pi^2}\left[\frac{g'^2}{2}\frac{M_1}{\mu}\left(-H_2(v_{\tilde{L}_i},x_1)+2H_2(v_{\tilde{e}_i},x_1)\right)-g'^2\frac{\mu}{M_1}H_2(w_{\tilde{L}_i},w_{\tilde{e}_i})\right], \quad (2a)$$

$$\epsilon_i^W = \frac{1}{16\pi^2}\frac{3g^2}{2}\frac{M_2}{\mu}H_2(v_{\tilde{L}_i},x_2)\,. \quad (2b)$$

There is no ϵ_i^G for the leptons, because they do not participate in the strong interaction and the contribution equivalent to ϵ^y is suppressed by the heavy mass scale of the right-handed neutrinos in the seesaw mechanism.

The function H_2 is defined as

$$H_2(x,y) = \frac{x\ln x}{(1-x)(x-y)} + \frac{y\ln y}{(1-y)(y-x)}. \quad (3)$$