

I Friedmann Folgen - Hubble Exp.

1, FG

- allgem. Rel. theorie
verknüpft Metrik der
Raumzeit mit E -Dichte

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

E wird vom
Energieinhalt
bestimmt

- Kosmologische Prinzip
Universum ist
- homogen
- isotrop

→ Metrik hat zwei
Parameter

Skalenspar. $R(t)$

Krümmung k $(+1, -1, 0)$

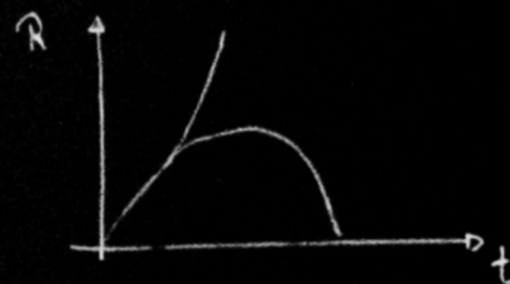
$$1, FG: \frac{\dot{R}^2}{R^2} + \frac{kc^2}{R^2} = \frac{8\pi G}{3} \rho$$

$$\text{Dichte: } \rho = \rho_m + \rho_r + \rho_\Lambda$$

Materie Strahlung Kosmol. Konstante

$$\rho_m \sim \frac{1}{R^3}, \quad \rho_r \sim \frac{1}{R^4}$$

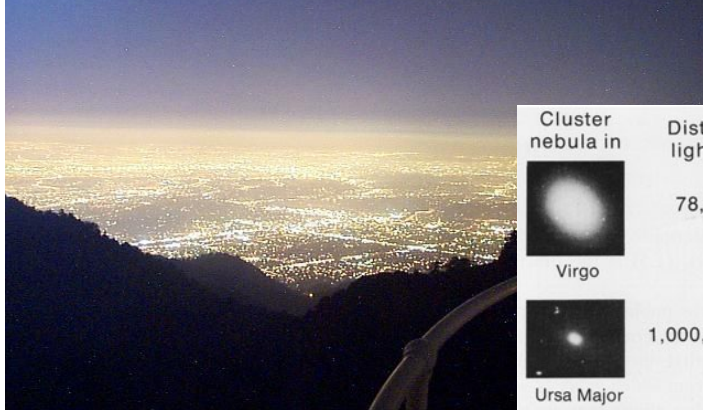
$$\rho \sim \frac{1}{R^n}, \text{ flaches Universum } k=0 \quad \begin{matrix} 2/3 \\ 2/n \end{matrix} \quad \begin{matrix} \sim t \\ \sim \sqrt{t} \end{matrix} \quad \begin{matrix} \text{Materie} \\ \text{Strahlung} \end{matrix}$$



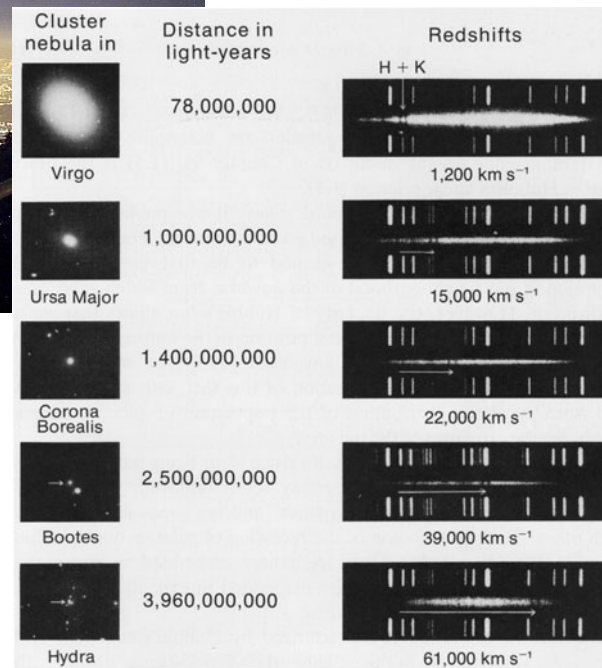
$$2, FG: \ddot{R} = -\frac{4\pi}{3} G R \left(\rho + 3 \frac{p}{c^2} \right)$$

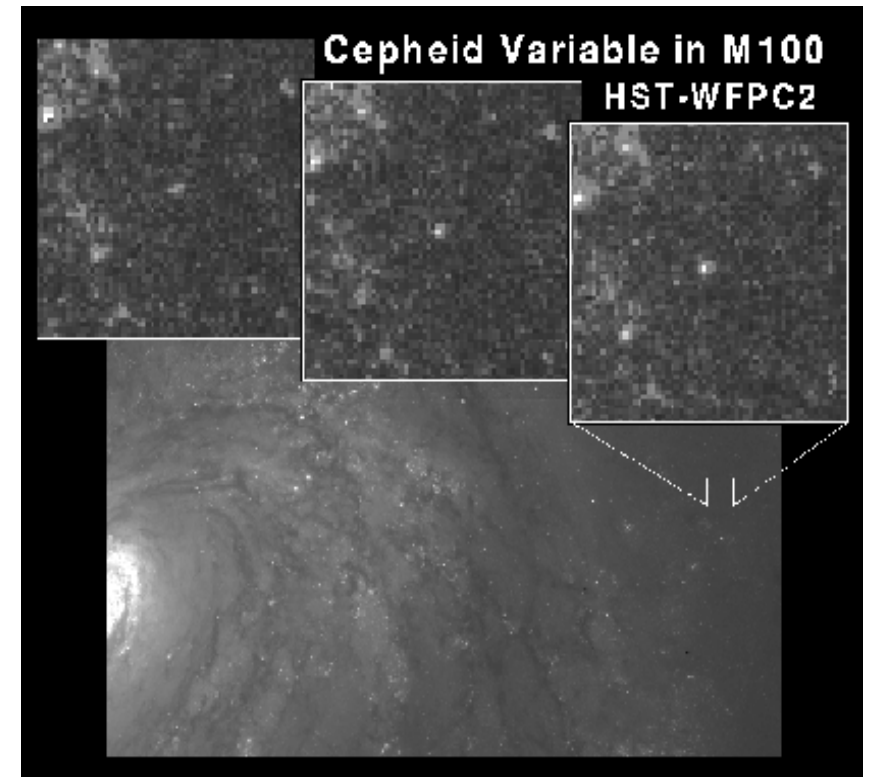
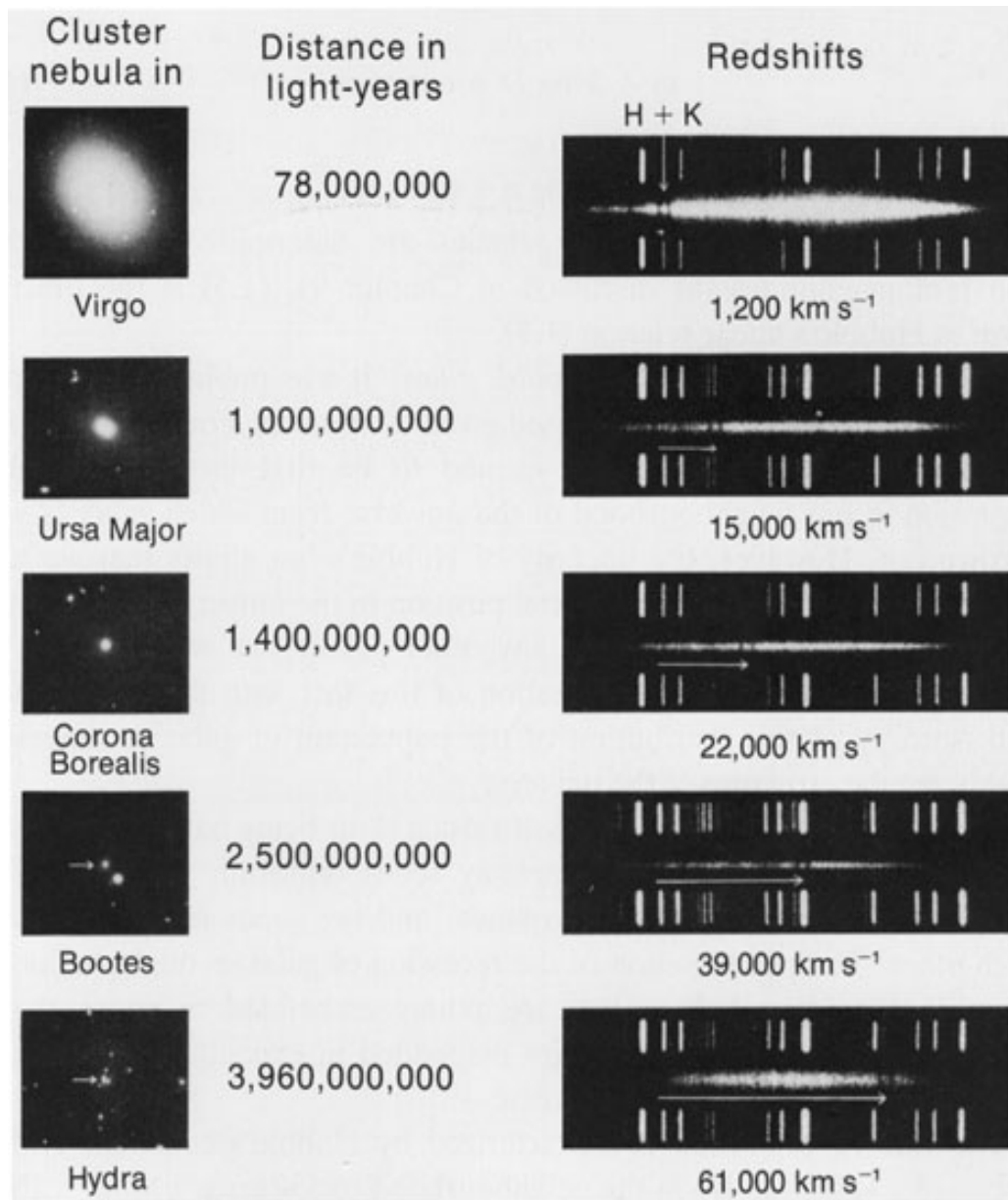


E.Hubble
1929

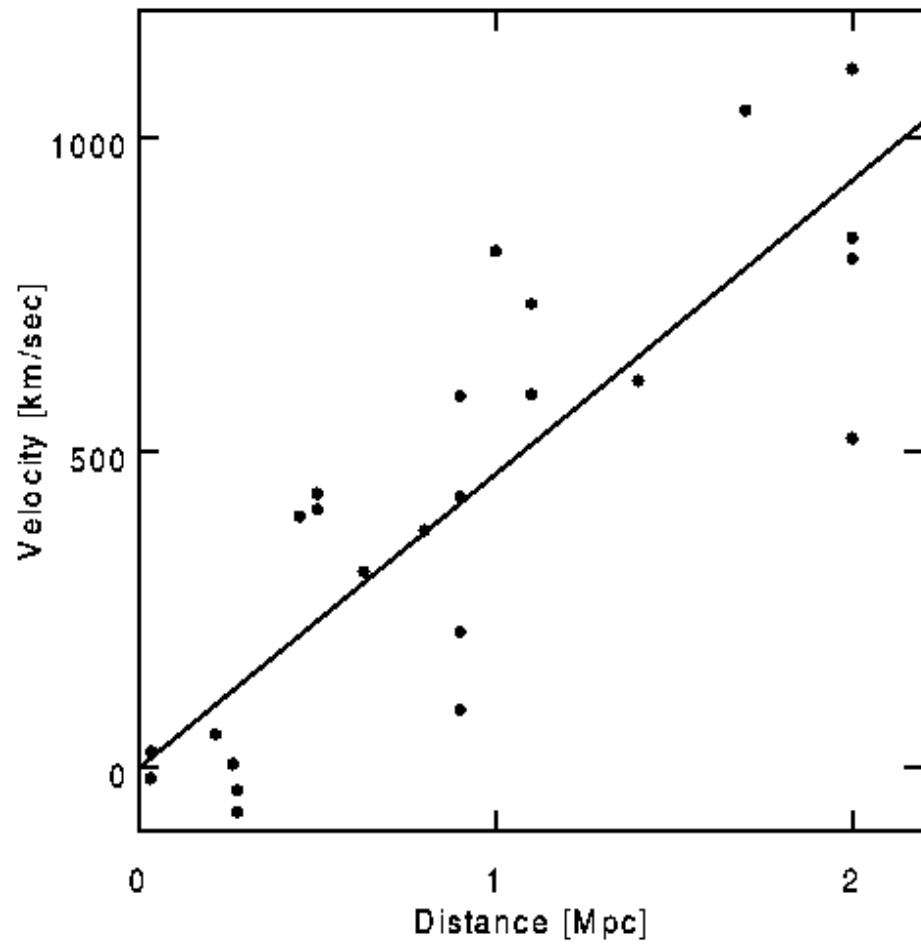


Mt. Wilson
Observatory



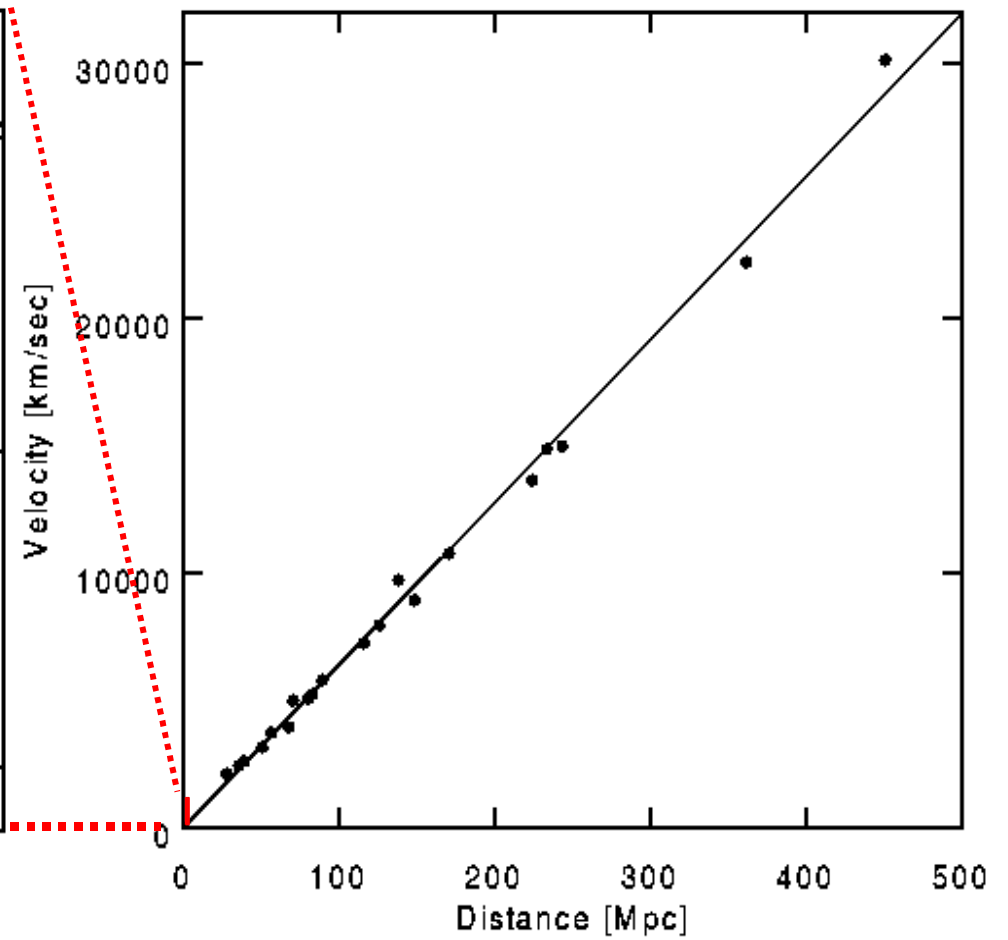


Hubble 1929

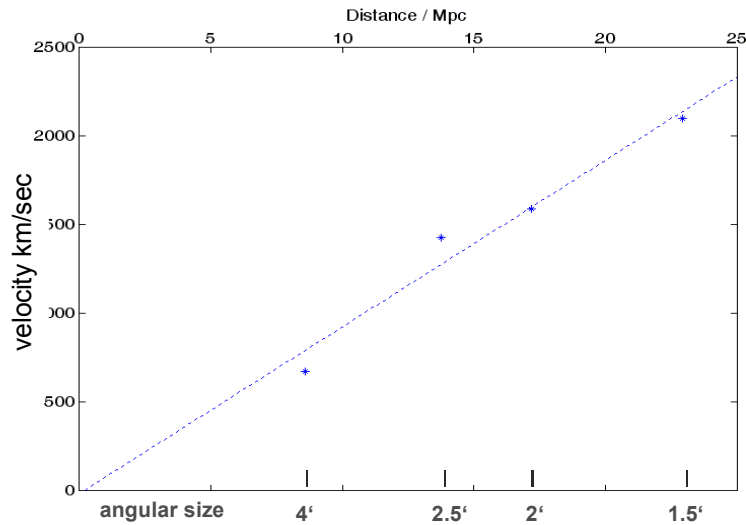
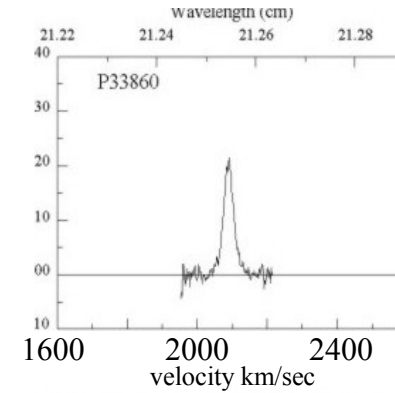
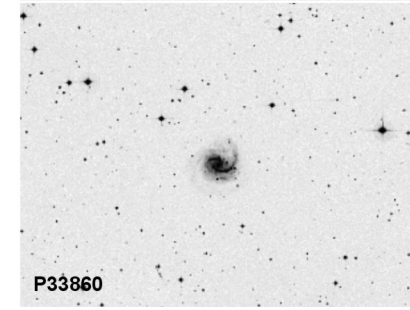
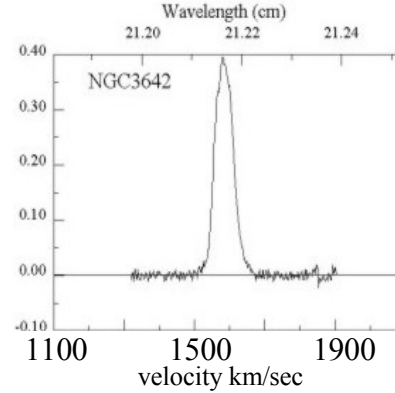
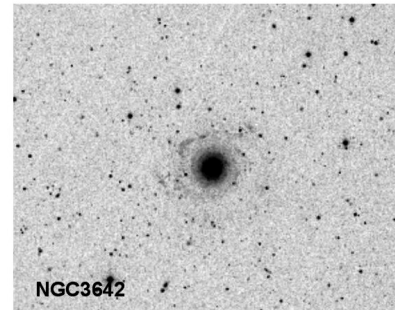
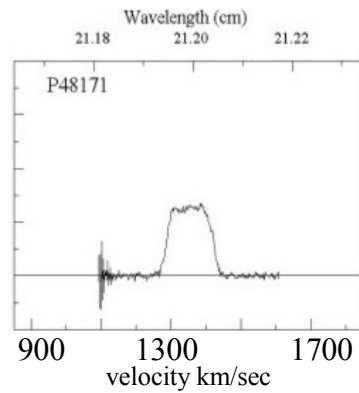
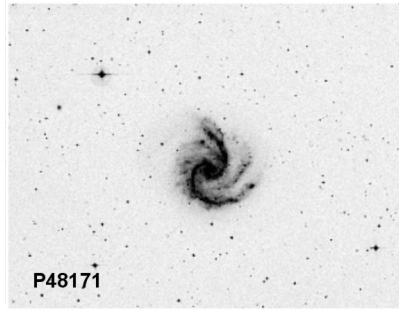
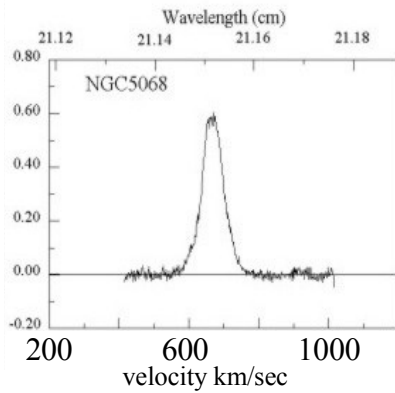
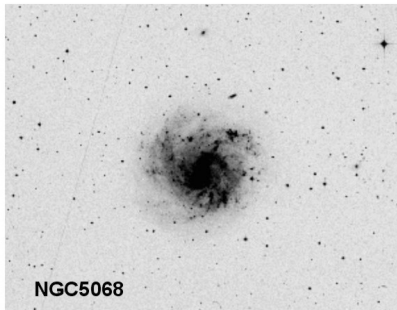


$H \sim 500 \text{ km/s/Mpc}$

1996 SN Ia

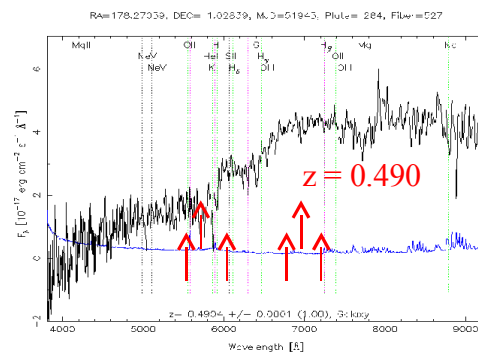
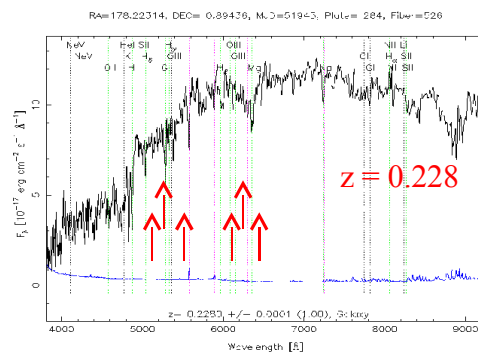
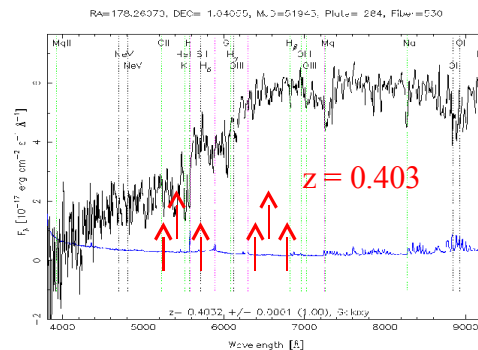
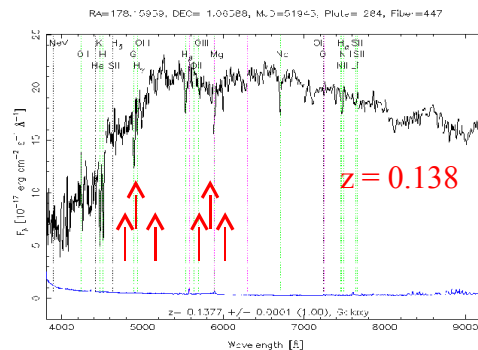
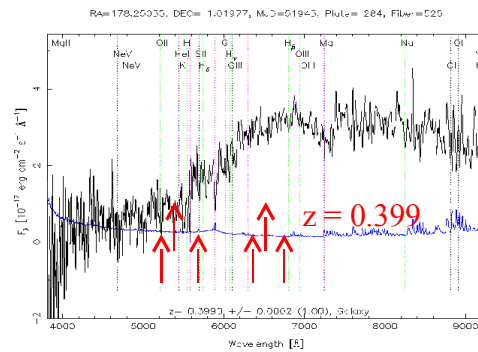
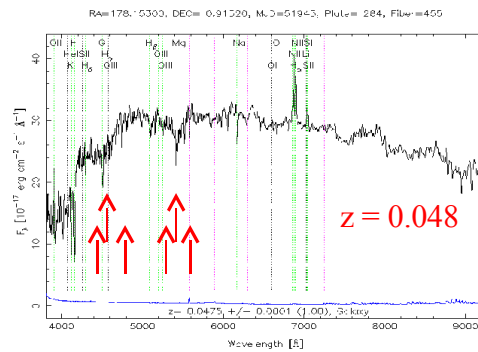
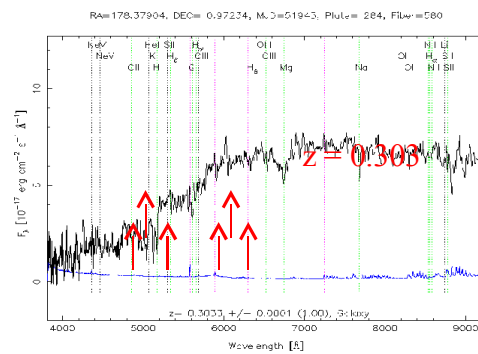
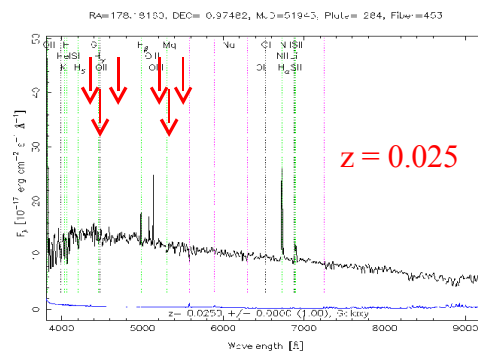


$H \sim 70 \text{ km/s/Mpc}$



21 cm Radio- Linie
scheinbare Größe der Galaxie
=> je kleiner (weiter) desto mehr rotverschoben

aus Annahme Galaxienradius = 10kpc
=> Abstand
=> Steigung ~ Hubble Parameter ~ 90 km/s/Mpc



2. Hubble Ausdehnung

1929 Hubble am Mt. Wilson Obs.

Rotverschiebung? entfernter Galaxien

$$z = \frac{\lambda' - \lambda}{\lambda} = \frac{v}{c}$$

Doppler Effekt $\lambda' = \lambda \sqrt{\frac{1+\beta}{1-\beta}}$, $\beta = \frac{v}{c}$

Hubble Gesetz $v = H \cdot r$
 Geschw. $\uparrow$ Abstand $\leftarrow$

$$H = (70 \pm 10) \text{ km/s/Mpc}$$

$$1 \text{ Mpc} = 10^6 \cdot 3,26 \text{ ly} \\ = 3,09 \cdot 10^{13} \text{ km}$$

$$\text{Andromeda } 2 \cdot 10^6 \text{ ly} \rightarrow 0,6 \text{ Mpc}$$

$$v = \frac{v}{c} = \frac{H}{c} \cdot 0,6 \text{ Mpc} = 1,6 \cdot 10^{-4}$$

$$H = \dot{R}/R$$

$$\text{kritische Dichte } \rho_c = \frac{3H^2}{8\pi G}$$

$$\Omega = \rho/\rho_c; \Omega = \Omega_m + \Omega_\Lambda + \Omega_\Lambda$$

$$\rho_c = \frac{5,1 \text{ GeV}}{\text{m}^3} \approx \frac{5 \text{ protonen}}{\text{m}^3}$$

$$H^2 G = \frac{kc^2}{H^2 R^2} = \Omega - 1$$

$$\Omega > 1 \Rightarrow k > 0$$

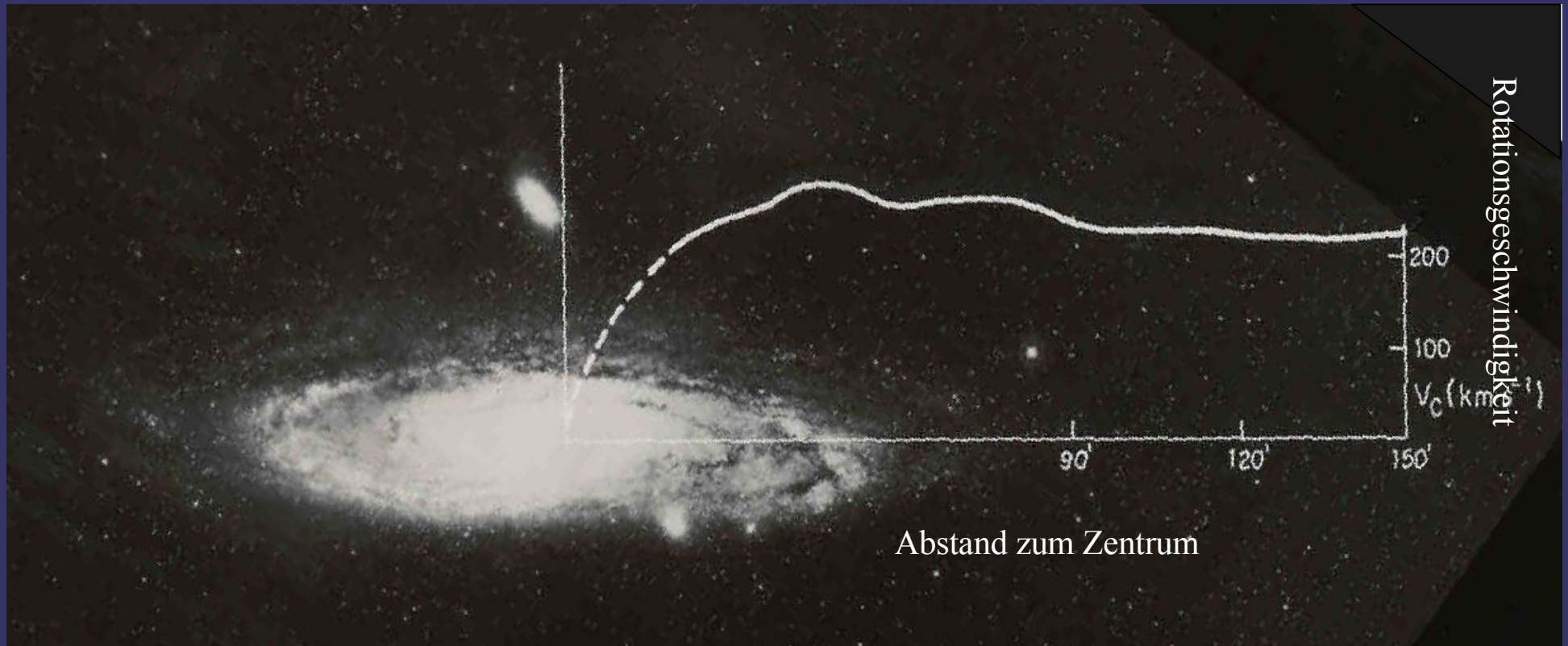
geschlossen

$$\sum \alpha > 180^\circ$$

$$\Omega = 1 \Rightarrow k = 0 \text{ flach } \sum \alpha = 180^\circ$$

$$\Omega < 1 \Rightarrow k < 0$$

offen $\sum \alpha < 180^\circ$



II Materie Dichte Ω

$$\Omega_{vis} < 0.01$$

$$\Omega_m = \frac{\rho_m}{\rho_c} = \frac{(M/L) \cdot L^3}{\rho_c} = \frac{M/L}{1000 M_\odot/L_\odot}$$

mittlerer Leuchtkoeff. $\downarrow_B = \frac{1.3 \cdot 10^3 L_\odot}{M_{pc^3}}$

a) Solare Umgebung $M/L = 5 M_\odot/L_\odot$
 $0.005 \ll 1$

b) Galaxie Rotationskurven

F. Zwicky 1930

$v(r)$ r : Abstand zum
Zentrum

2.1 cm Linie von H_2

$$F_{Zentr} = F_{Gav} \Rightarrow m \frac{v^2(r)}{r} = G \frac{m M(r)}{r^2}$$

$$\Rightarrow v(r) = \sqrt{G \frac{M(r)}{r}}$$

II Materie Dichte Ω

$$\Omega_{vis} < 0.01$$

$$\Omega_m = \frac{\rho_m}{\rho_c} = \frac{(M/L) d_B}{\rho_c} = \frac{M/L}{1000 M_\odot/L_\odot}$$

$$\text{mittler Leuchtkraft } L_B = \frac{1.3 \cdot 10^8 L_\odot}{M_{pc^3}}$$

$$a, \text{ solare Umgebung } M/L = 5 M_\odot/L_\odot$$

$$\Rightarrow \Omega_{m \text{ solar}} = 0.005 \ll 1$$

b, Galaxie Rotationskurven

Fr. Zwicky 1930

$v(r)$ r : Abstand zum Zentrum

21cm Linie von H_2

$$F_{\text{Zentr}} = F_{\text{Gru}} \Rightarrow m \frac{v(r)^2}{r} = G \frac{m M(r)}{r^2}$$

$$\Rightarrow v(r) = \sqrt{G \frac{M(r)}{r}} \sim \frac{1}{\sqrt{r}} \quad M \text{ const.}$$

flache Rotationskurven

$\Rightarrow$ Halbe Dunkler Materie

$$M/L = (20-50) M_\odot/L_\odot$$

$$\Rightarrow \Omega_{m, \text{Halo}} = 0.02 - 0.05 > \Omega_{vis}$$

lokale Dichte

$$\sim 0.3 \text{ GeV/cm}^3$$

$$= 3 \cdot 10^5 \text{ GeV/m}^3$$



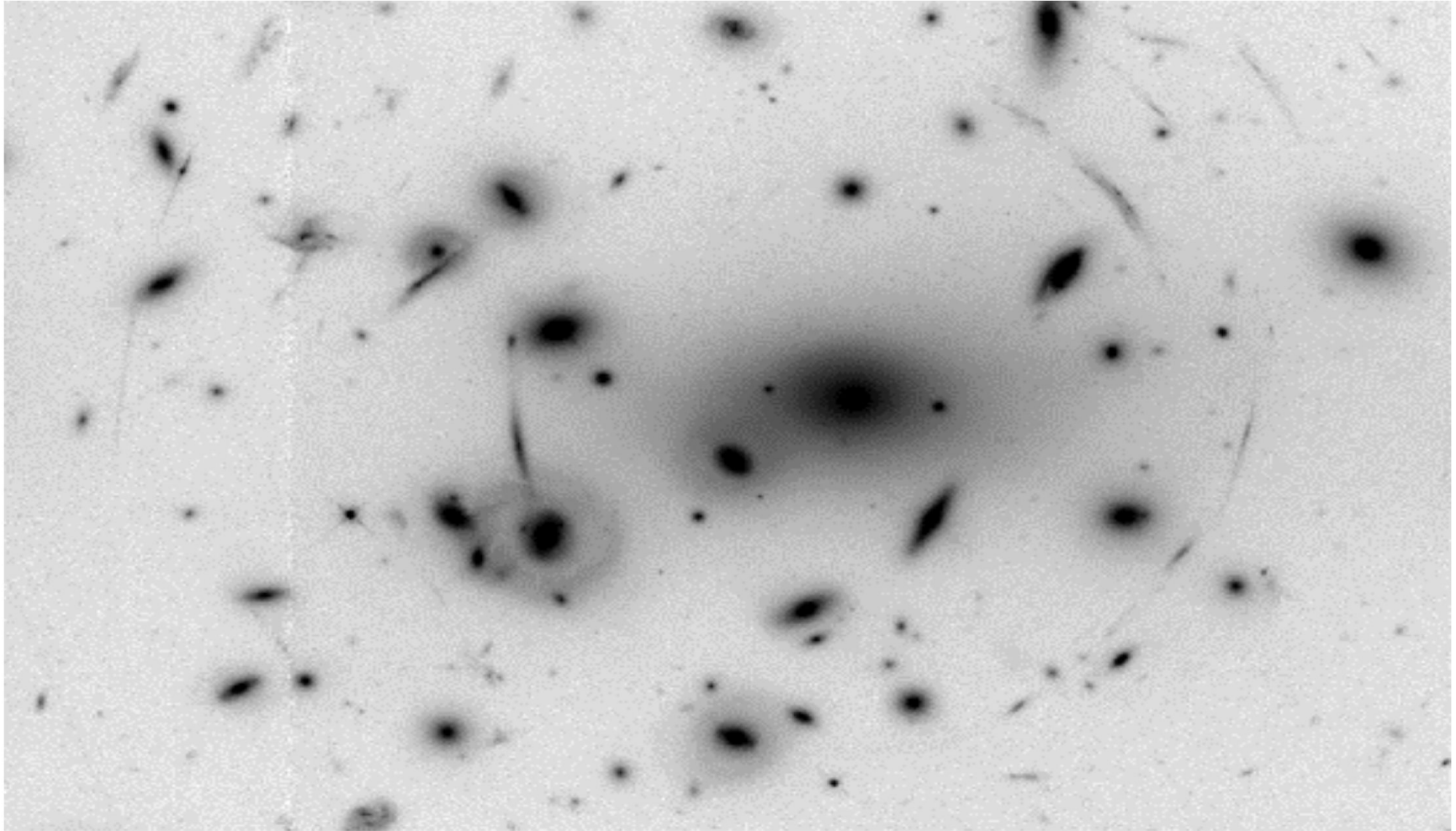


Gravitational Lens
Galaxy Cluster 0024+1654

HST • WFPC2

PRC96-10 • ST ScI OPO • April 24, 1996

W.N. Colley (Princeton University), E. Turner (Princeton University),
J.A. Tyson (AT&T Bell Labs) and NASA

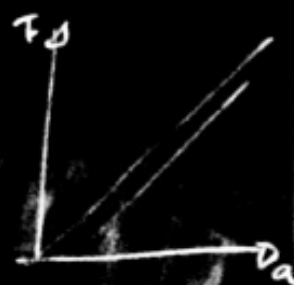


Gravitationspotential

$$\frac{GM}{r} \rightarrow \frac{GM}{r} + V(\xi_0)$$

→ Einführung einer Skalarfeld

→ verschiedene ξ_0 nötig



Modifizierte Newtonsche Dyn (MOND)

$$F = ma \quad \text{für } a > a_0 \rightarrow F_2 \propto \frac{1}{r^2}$$

$$F = m \frac{a^2}{a_0} \quad \text{für } a < a_0 \rightarrow F_2 \propto \frac{1}{r} \sim \frac{1}{r^2} = F_G$$

$$\Rightarrow \forall v_{\infty} = \text{const} ; a_0 \propto c^2$$

c, Galaxien Cluster

$$M/L \approx (210 \pm 70) M_\odot/L_\odot$$

$$\Omega_{m, \text{cluster}} = \underline{\underline{0.21 \pm 0.07}}$$



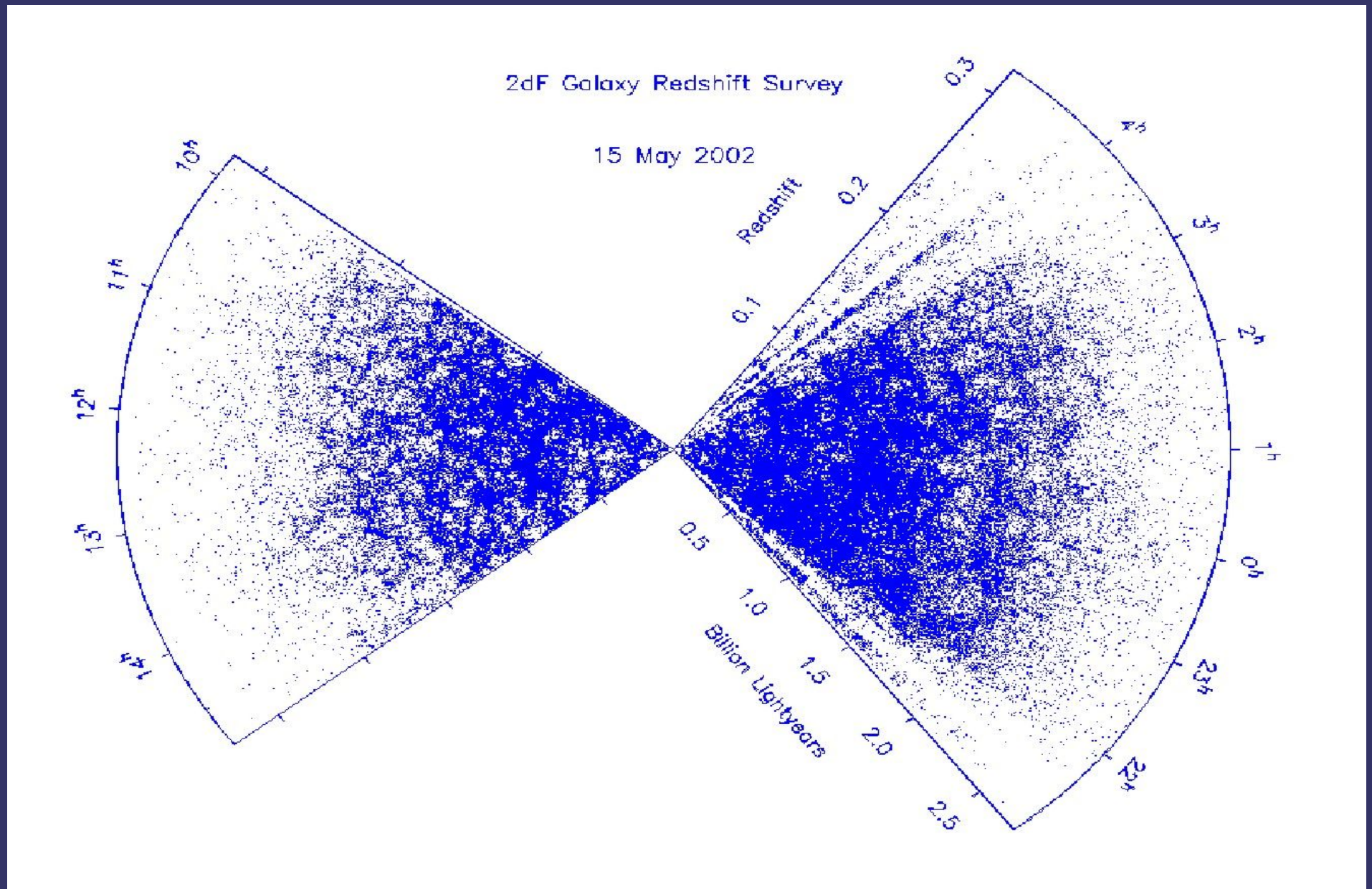
Einstein Ring $\propto \sqrt{M}$

d, Peculiar Geschwindigkeit

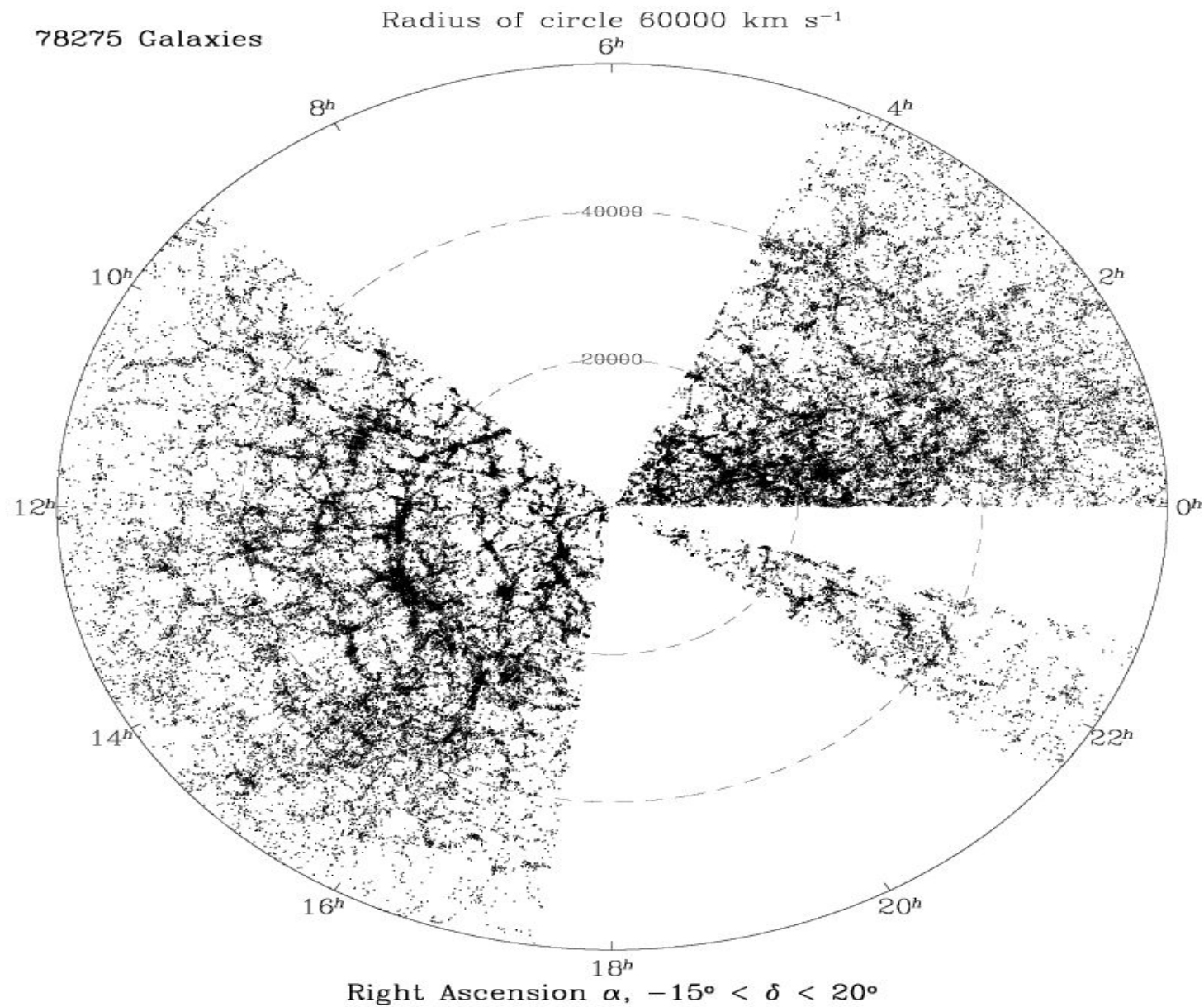
$$v_{\text{pec}} \approx (627 \pm 22) \text{ km s}^{-1}$$

lokale Überdichte $\frac{\delta \rho}{\rho}$

$$\Rightarrow \Omega_m \approx \underline{\underline{0.3-0.8}}$$

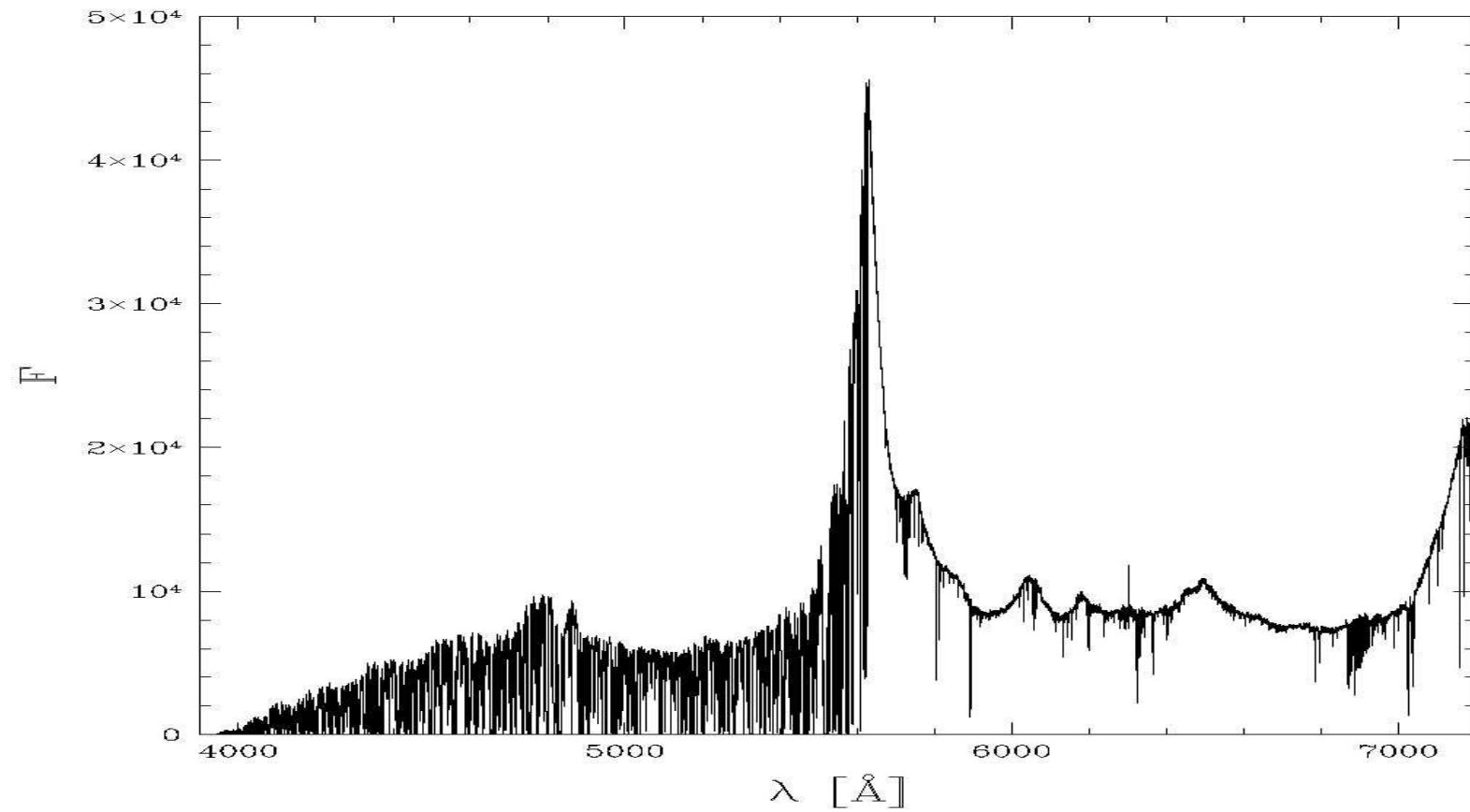


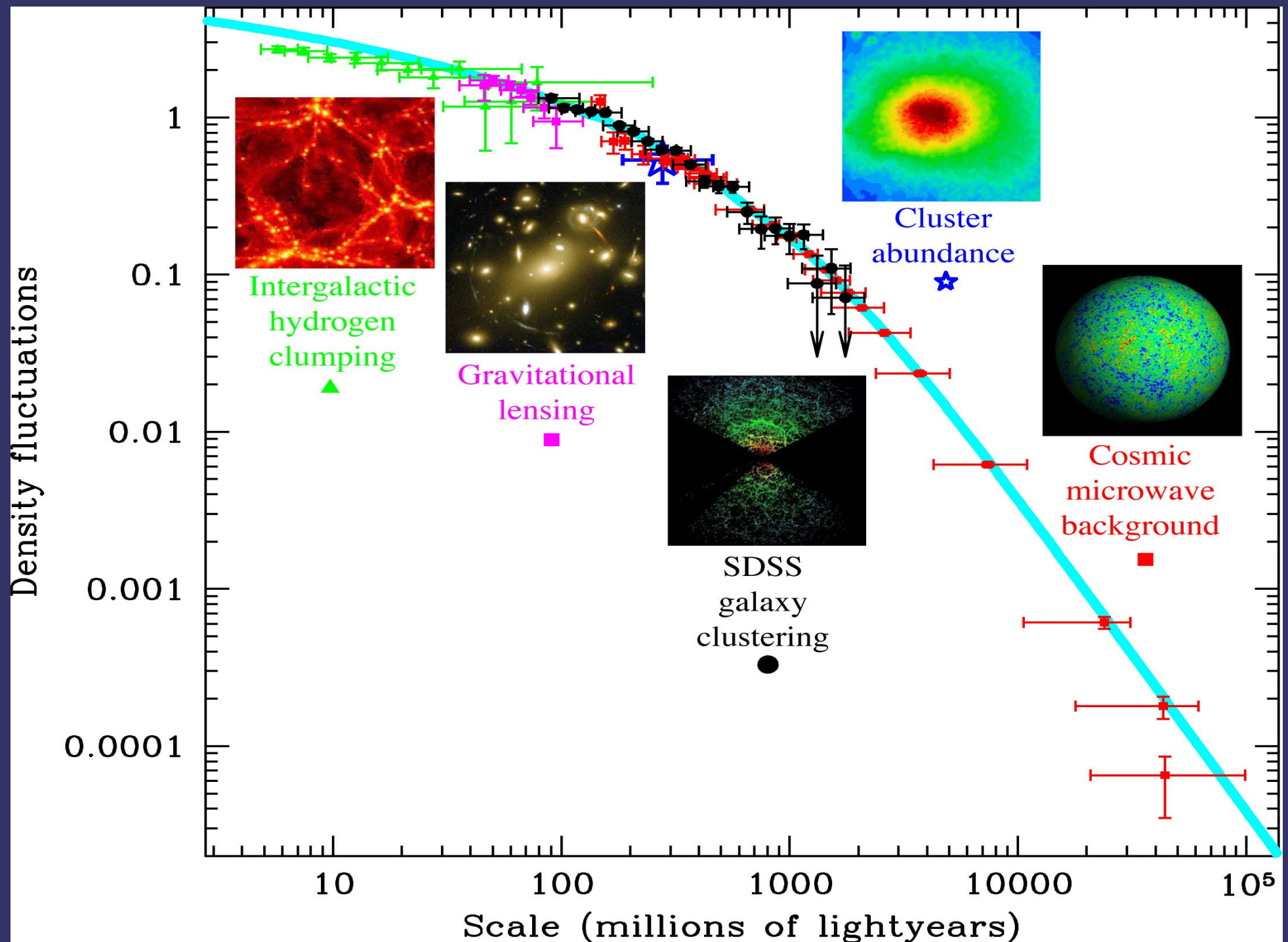
Sloan Digital Sky Survey

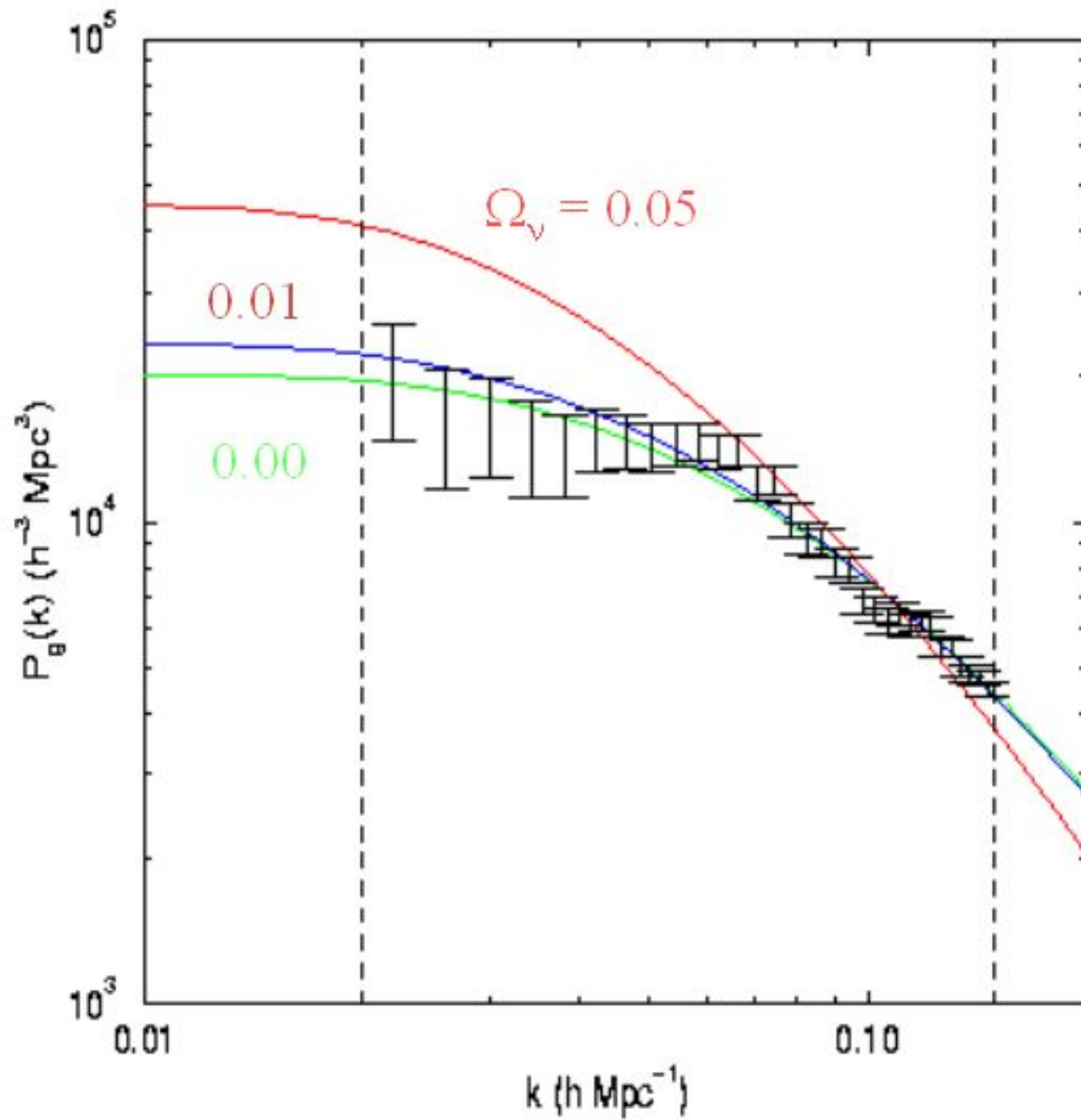




Lyman α Forest







II Materie Dichte Ω

ab Strukturbildung
Power-Spektrum

Wachstum von Dichte fluktuationen
 $\frac{\delta \rho}{\rho}$

$$\Rightarrow \Omega_m \sim 0.3$$

$$\frac{\Omega_{HDM}}{\Omega_{CDM}} \text{ max } 15\%$$



L

Kollaps Zeit $\sim$ free fall time $t_{\text{ff}} \neq L$

$$\text{Jeans Länge } \lambda_J = t_{\text{ff}} v_s$$

$L \ll \lambda_J \rightarrow$ Dichte oszillationen
kein $\frac{\delta \rho}{\rho}$ Wachstum

$L \gg \lambda_J \rightarrow$ Kollaps

$$v_s \sim \frac{kT}{m_{\text{auskühlen}}}$$

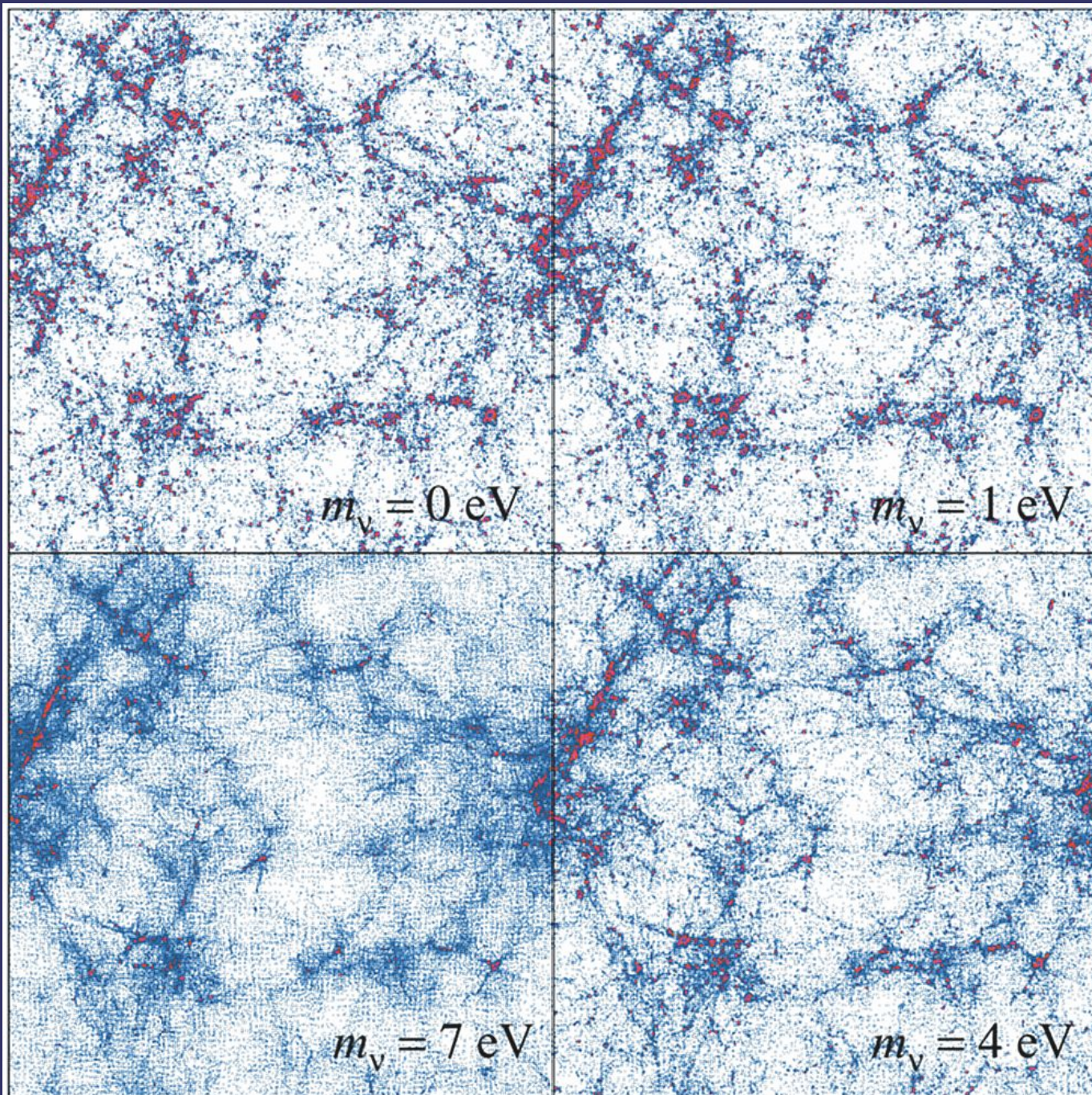
$\sim c$ relativistische Teilchen

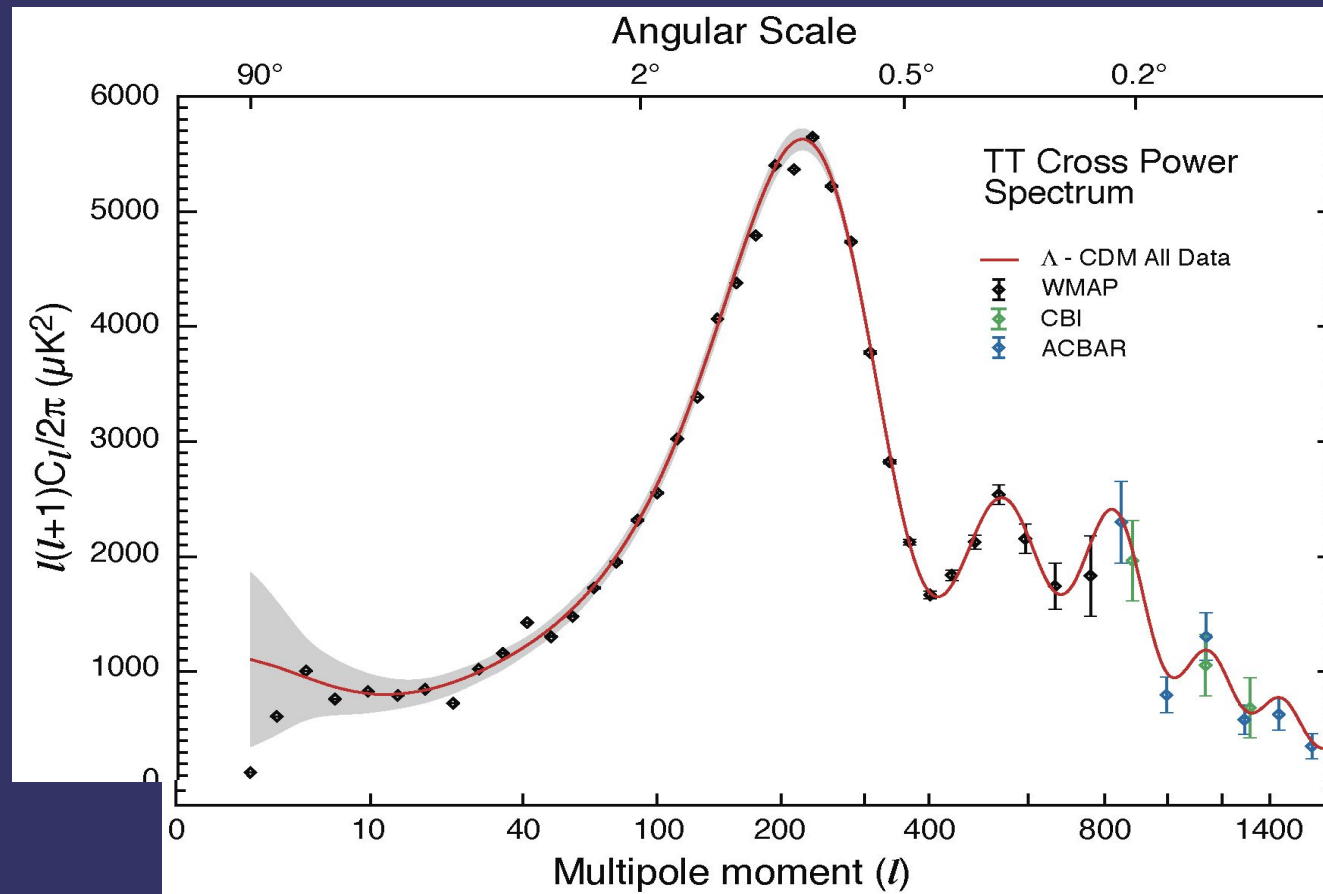
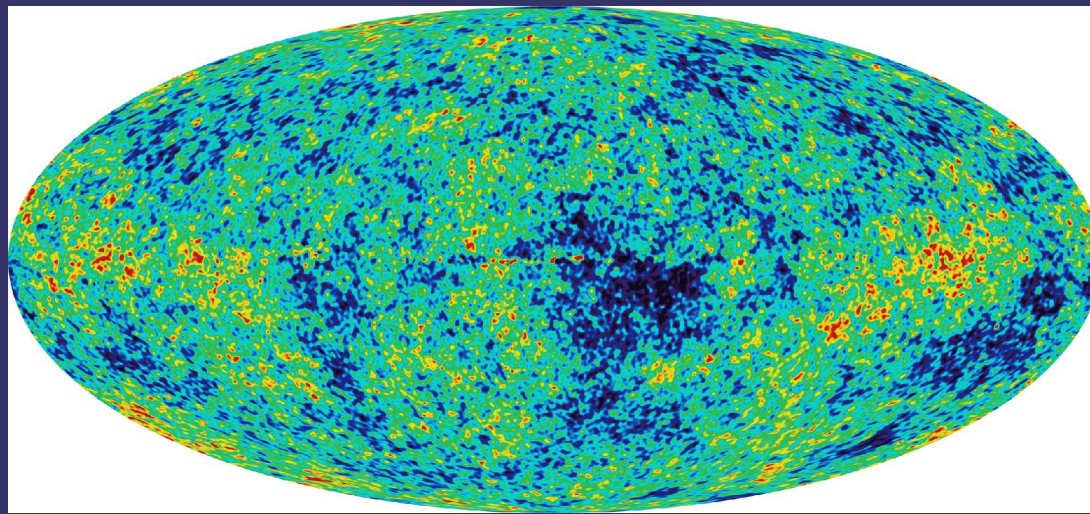
hot dark matter HDM

λ_J groß

$\rightarrow$ Wachstum auf kleinen
Skalen unterdrückt

cold dark matter CDM





7 Neutrinos als DM

$$N_\nu \approx 113 \text{ cm}^{-3}$$

$$\Rightarrow \Omega_\nu = \frac{\sum m_\nu c^2}{47 \text{ eV}}$$

ν sind HDM

$$\Omega_m \approx 0,3$$

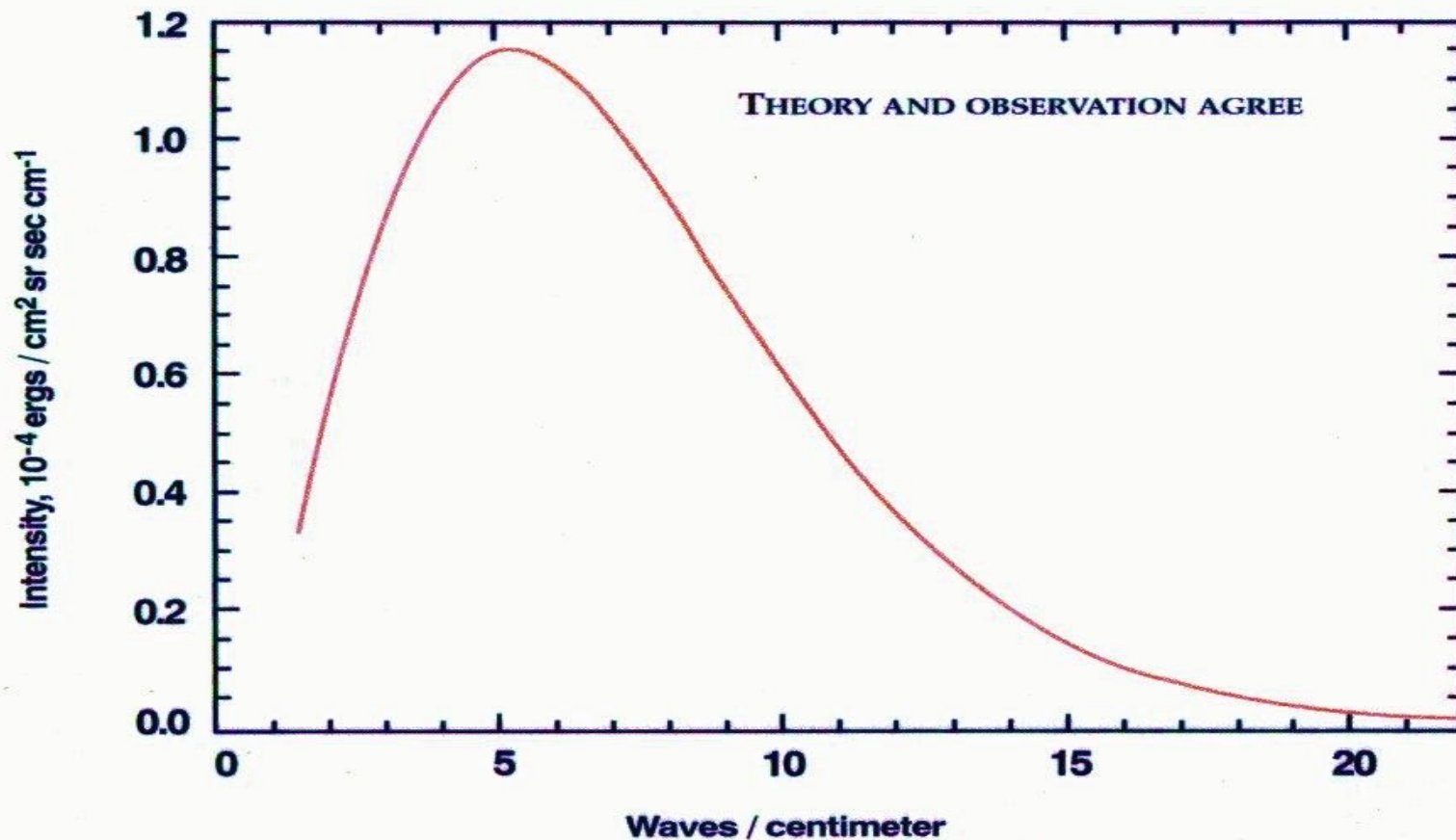
hauptsächlich CDM

max 15% HDM

COBE - FIRAS



COSMIC MICROWAVE BACKGROUND SPECTRUM FROM COBE



III CMB

1. 2,7 K Hintergrund

1965 entdeckt von
Penzias u. Wilson

zuerst von Gamow vorhergesagt

Anisotropie $10^{-5} \approx \frac{\delta T}{T}$

Anisotropie des CMB

a. Sachs-Wolfe Effekt

- Rotverschiebung wg. Grav. pot

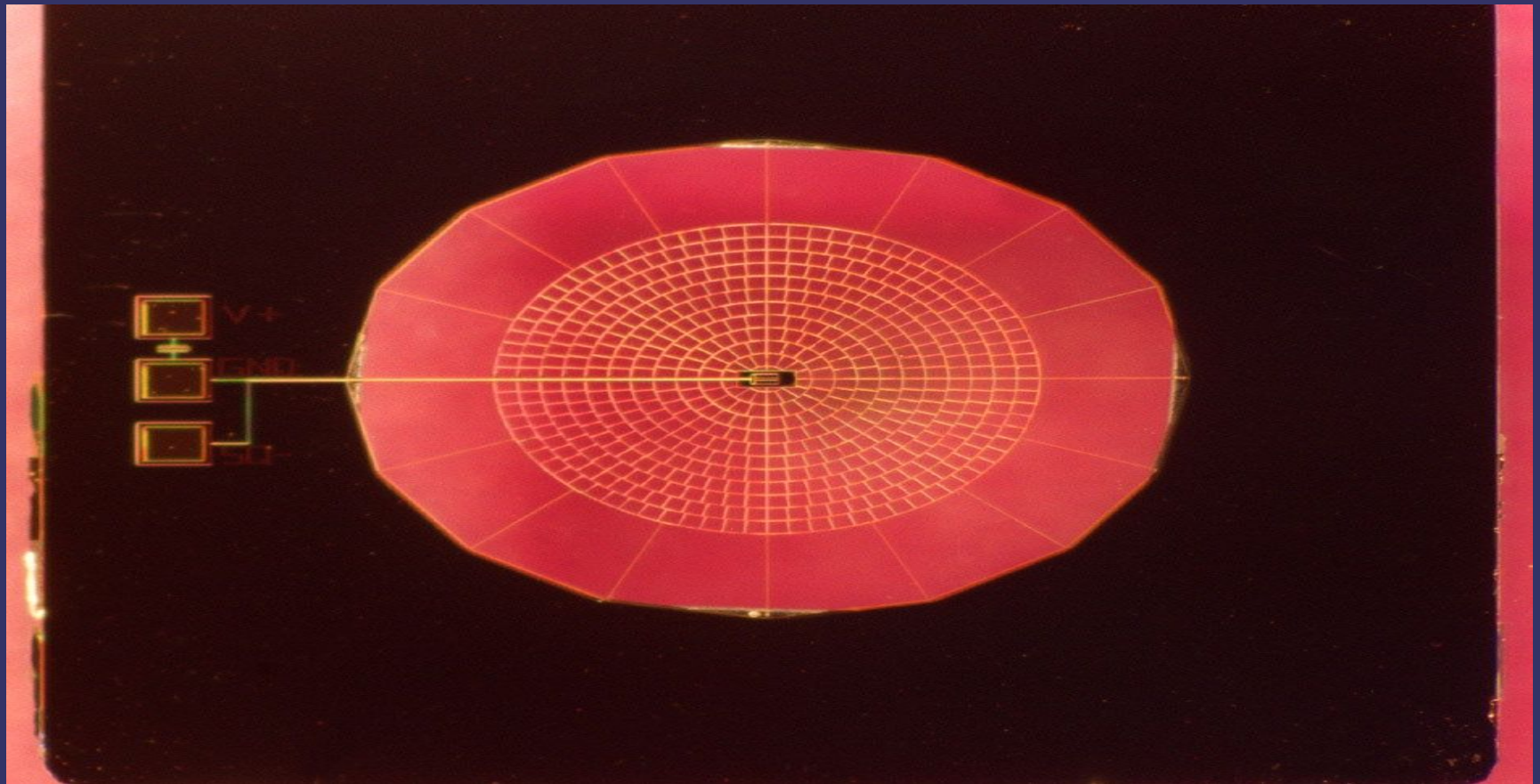


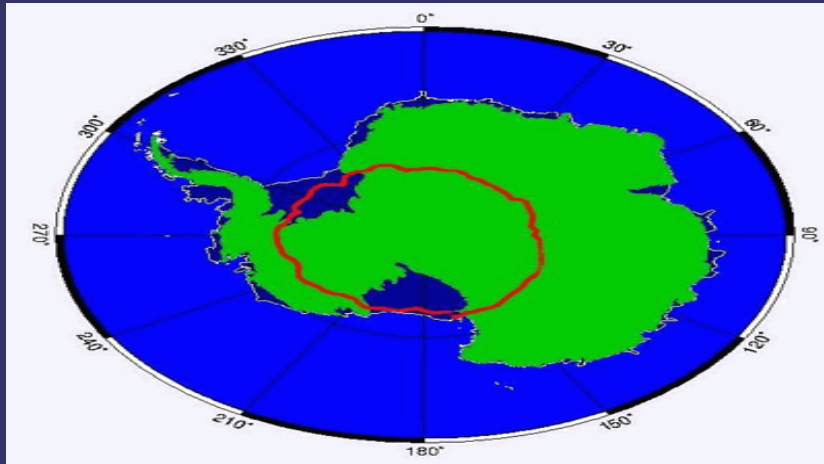
$$\frac{\delta T}{T} = \frac{\delta E}{E} = \frac{\delta \phi}{c^2}$$

- Blauversch. durch Zeitdilatation

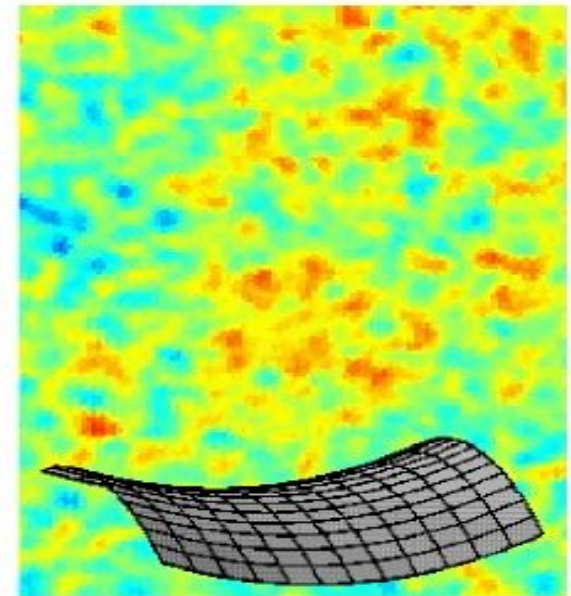
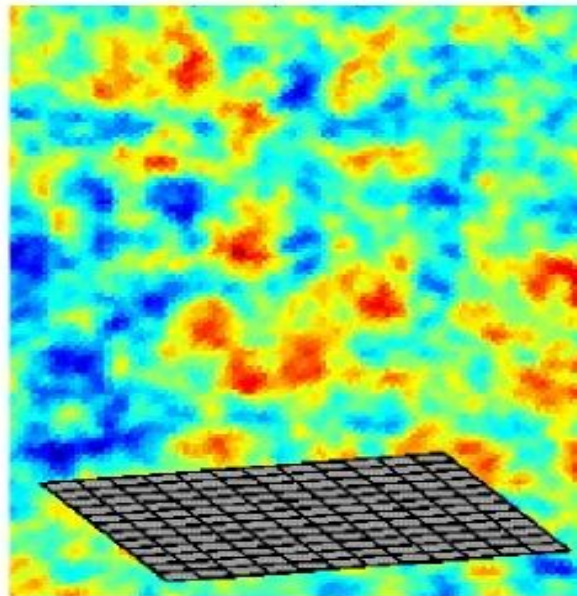
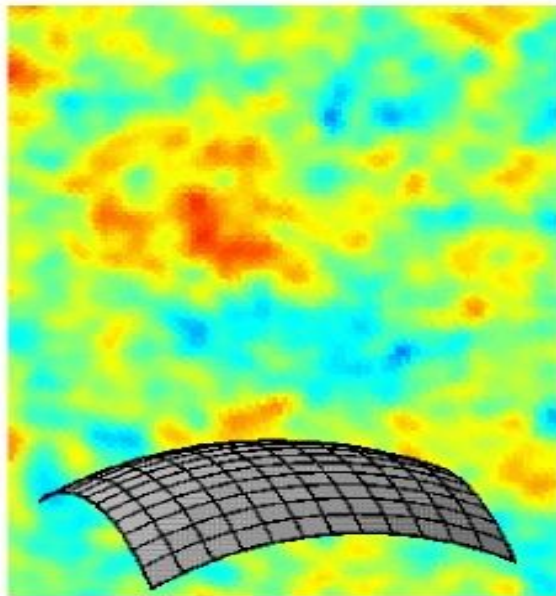
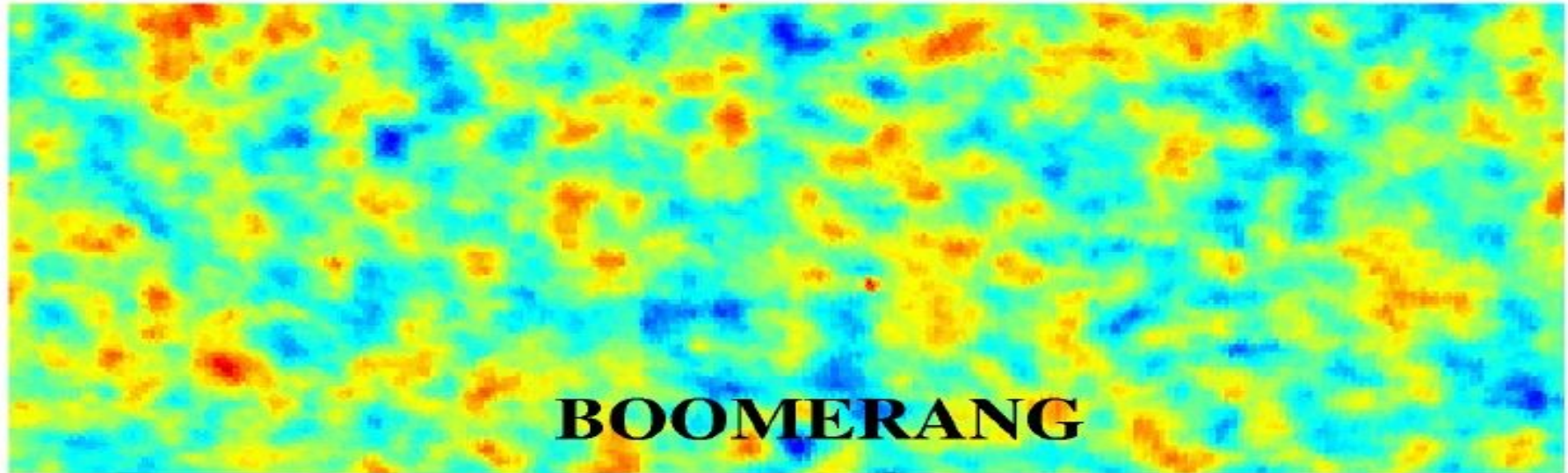
$$\frac{\delta T}{T}_{sw} = \frac{1}{3} \frac{\delta \phi}{c^2}$$

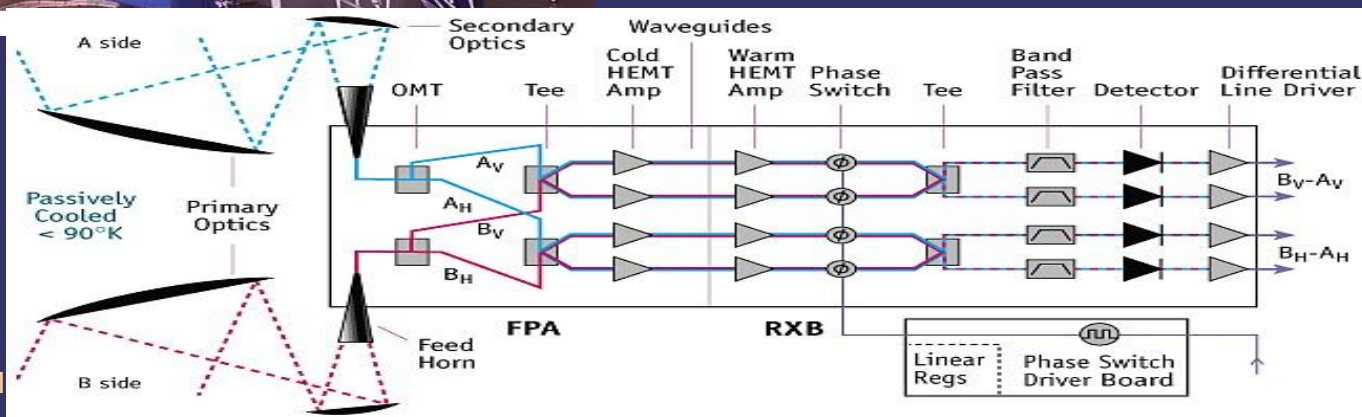
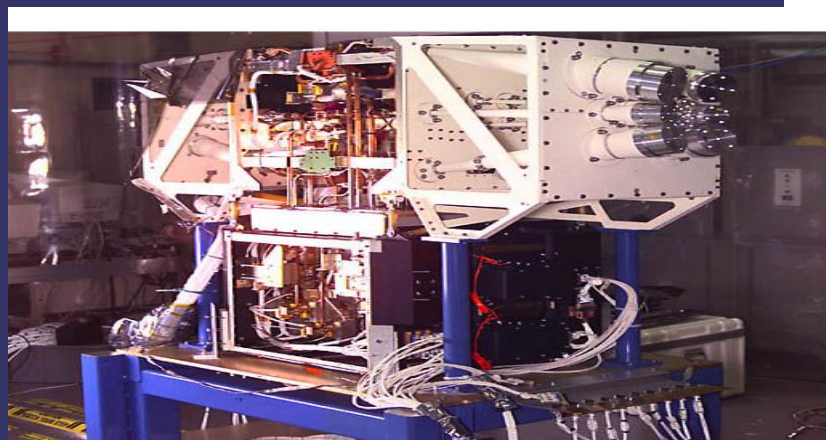
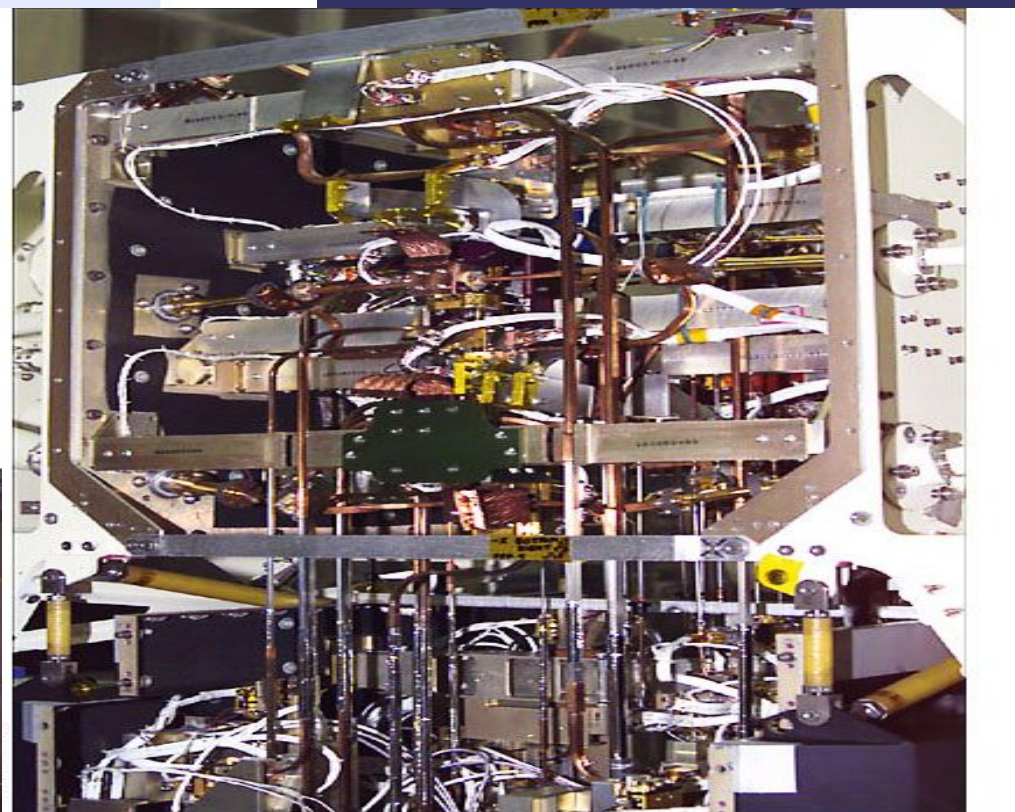
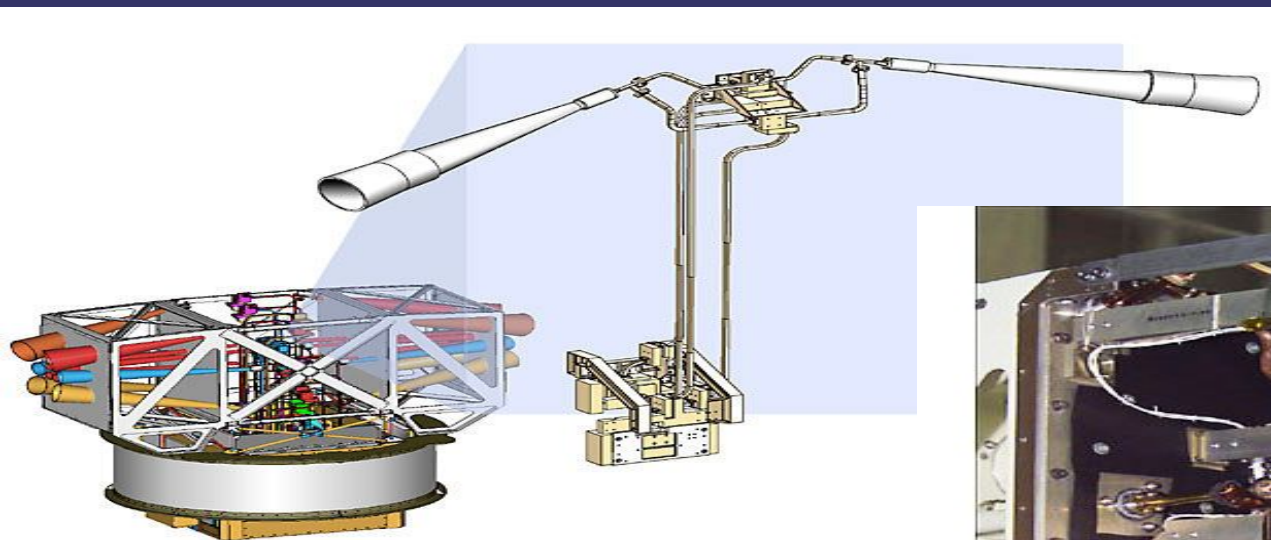
Boomerang





25°

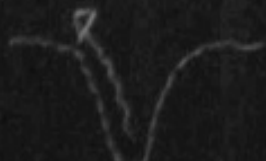




III CMB

Anisotropie

a) Sun-Effect



$$\frac{\delta T}{T} \approx \frac{1}{3} \frac{\delta \phi}{c^2}$$

b) Doppler Verschiebungen

$$\frac{\delta T}{T} = \frac{\delta \phi}{c}$$

$\delta \phi \sim$ Beschleunigung

$$\sim \dot{\phi} \delta t \approx k \delta \phi$$

$$\frac{\delta T}{T_{29}} \sim k \delta \phi$$

c) Adiabatische Fluide

kompresierte Regionen heißer

$$n_{\gamma} \sim T^3 \rightarrow \frac{\delta n_{\gamma}}{n} = \frac{1}{3} \frac{\delta n}{n} = \frac{1}{3} \frac{\delta \rho}{\rho} \sim \frac{1}{3} \delta \phi$$

$$\Delta \delta \phi = \sqrt{6} k$$

$$k^2 \delta \phi \sim \delta \rho$$

d) Silk-Dämpfung

Photonen diffundieren aus überdichteten Regionen hinaus

$$\rightarrow \text{freie Weglänge } \lambda_{\text{ph}}$$

dämpft
kl. Skalen

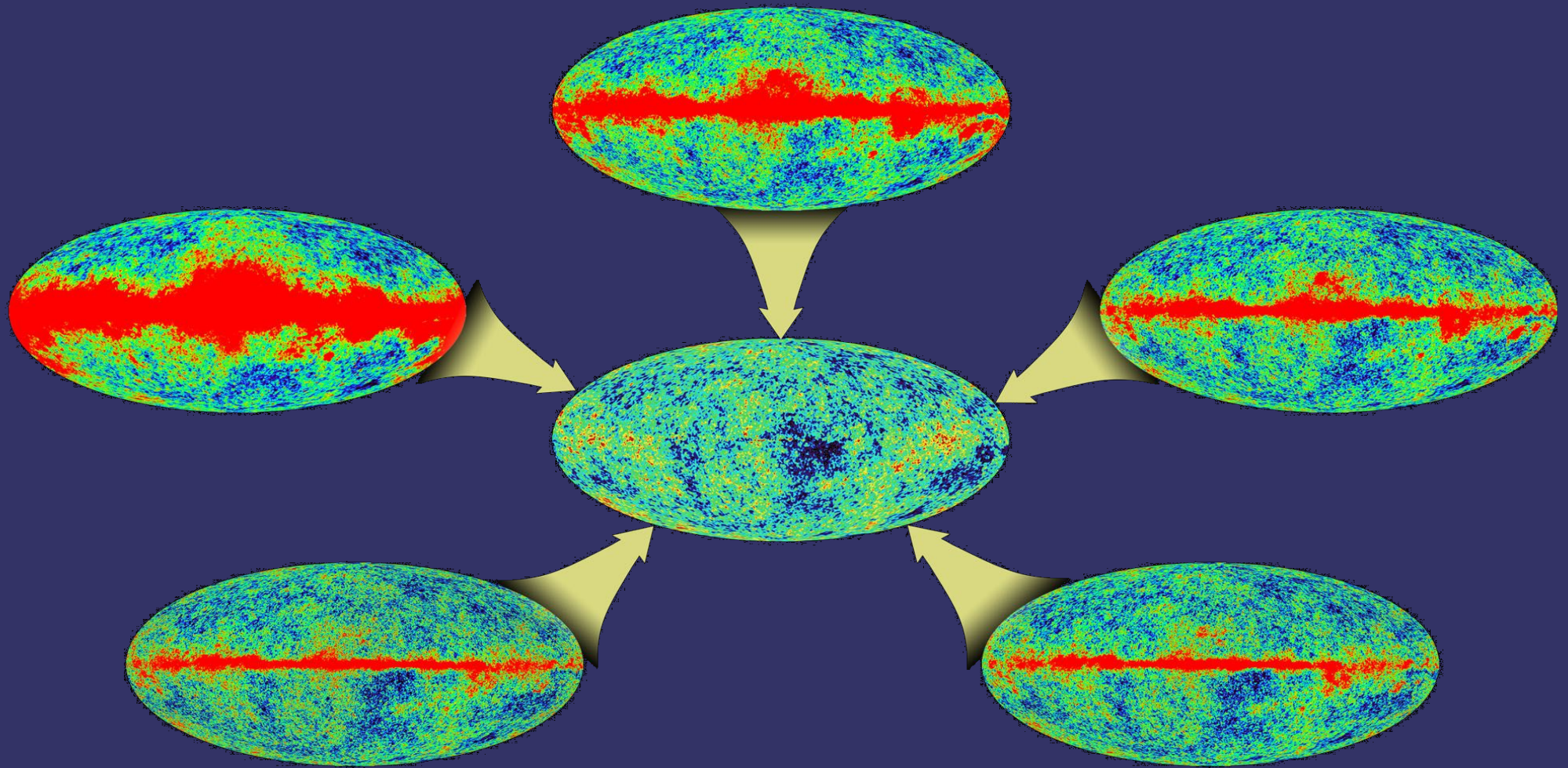
$$\rightarrow \Omega_D$$

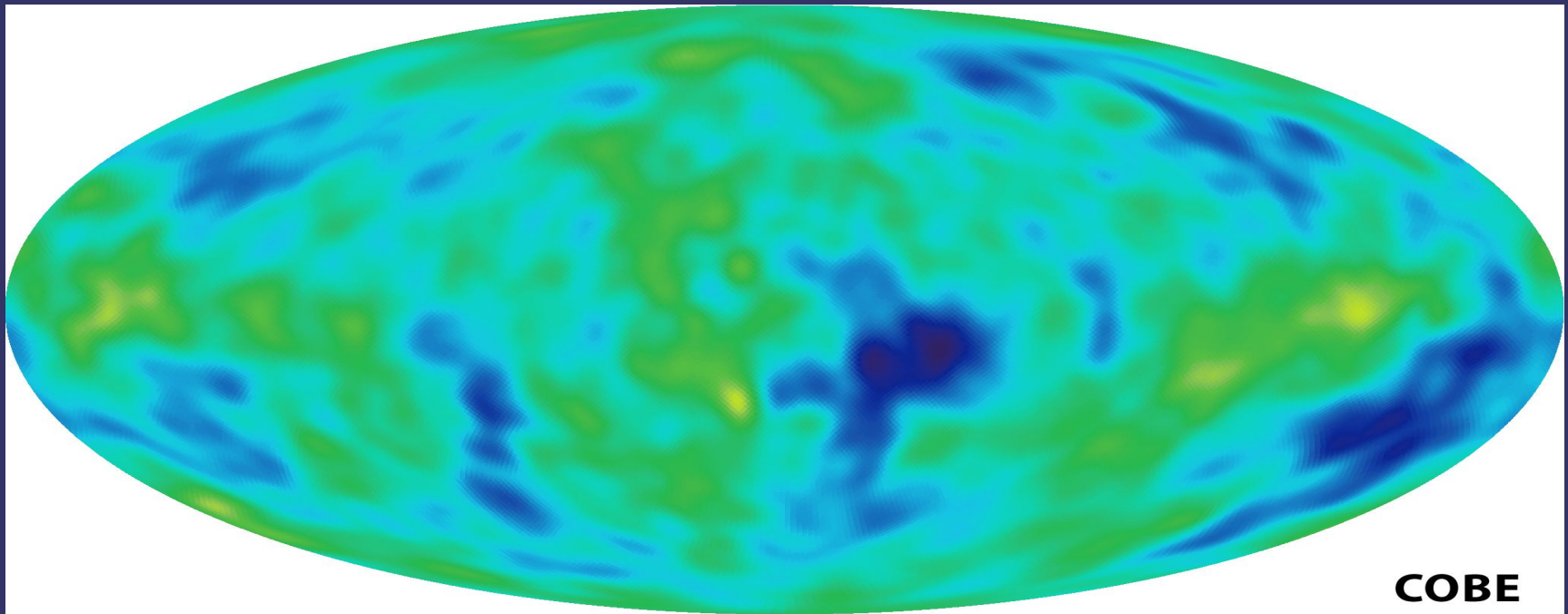
Photonen diffundieren aus überdichteten Regionen hinaus

$\rightarrow$ freie Weglänge Photonen

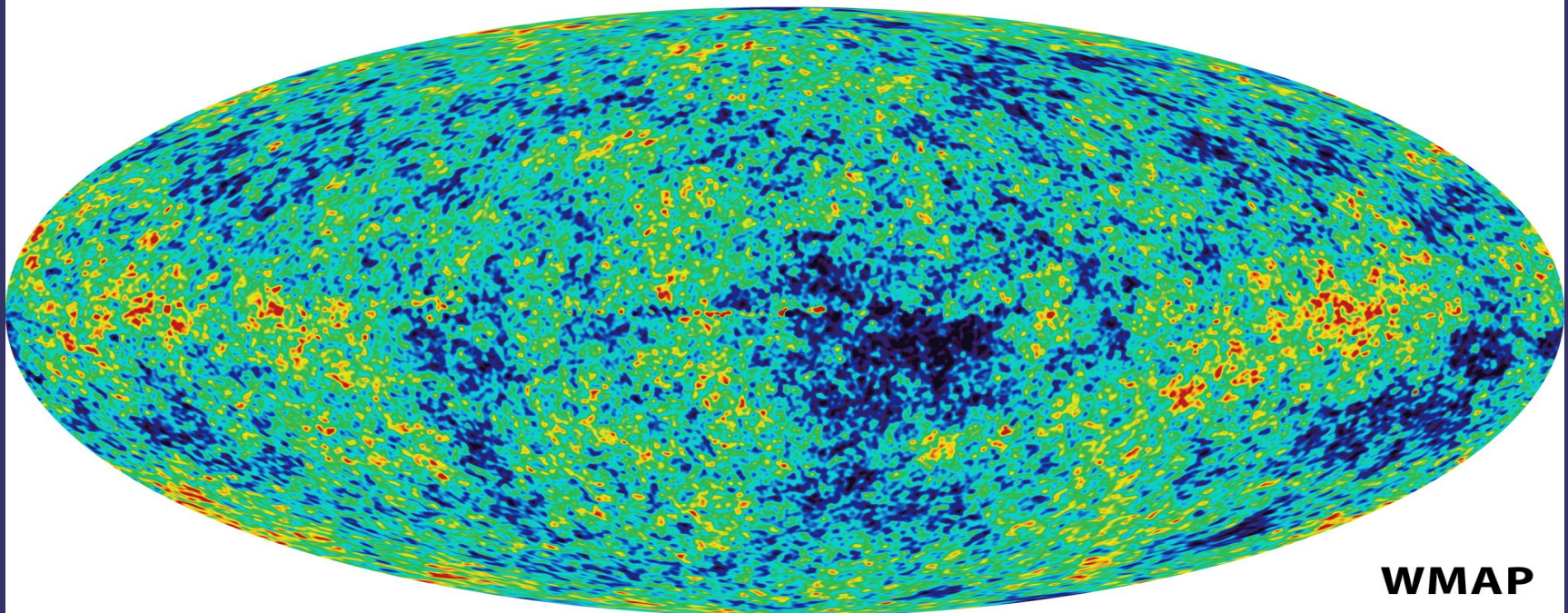
dämpft
kl. Skalen

$$\rightarrow \Omega_D$$





COBE



WMAP

